

Midterm 1

Tuesday, 2/27/2007
9:40-11:00 AM

Fundamental Constants:

Avogadro's number, N_A : 6.02×10^{23}

Gas Constant, R : $8.315 \text{ J/mol}\cdot\text{K}$

Boltzmann's Constant, k_B : $1.38 \times 10^{-23} \text{ J/K}$

Stefan-Boltzmann Constant, σ : $5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4$

1. (15 pts.) The volume coefficient β is defined as $\beta = (1/V) (dV/dT)$.

a) Calculate β as a function of temperature for the air in a hot balloon. Treat the air as an ideal gas. Assume that the air in the balloon does not escape and the heating process is slow enough to be considered quasistatic.

b) What is β at room temperature (20°C)?

2) (20 pts.) The density of atoms, mostly hydrogen, in interstellar space is about one per cubic centimeter. Assume that the diameter of a hydrogen atom is 10^{-10} m and the temperature of this gas is $\sim 3000 \text{ K}$.

a) Estimate the pressure of this gas and express the pressure in torr. (1 atm = 760 torr = $1.01 \times 10^5 \text{ Pa}$)

b) Estimate the mean free path.

3. (30 pts.) Let us consider a system in which every particle has only two accessible states. Suppose this system is in thermal contact with a reservoir at temperature T . State 1 has energy $E_1 = 0$ and state 2 has energy $E_2 = \varepsilon$.

a) What is the relative probability of finding a particle in state 1 versus in state 2 (*Hint*: use the Boltzmann factor to calculate $P(E_1)/P(E_2)$).

b) When properly normalized, what are $P(E_1)$ and $P(E_2)$?

c) What is the average energy of each particle?

d) What is the total internal energy of the system, if the total number of particles is N ?

e) Assuming the volume of this system does not change, calculate the heat capacity of the system, C_V .

4. (35 pts.) There are two boxes of ideal gas, each with n moles of gas molecules. Box 1 is at temperatures, T_1 and box 2 at T_2 . Now we bring them into thermal contact.

a) What is the final temperature when thermal equilibrium is reached?

b) Calculate the entropy change for box 1 and box 2. (Please specify the reversible path that you choose for this calculation.)

c) Calculate the entropy change of the combined system of box 1 and 2, ΔS_{total} . Show that $\Delta S_{\text{total}} \geq 0$.

d) The number of states for an ideal gas is

$$g(N, U) = KU^{3N/2} \quad (1),$$

where K is a constant, U , the internal energy and N , the number of molecules. Calculate the change in entropy for box 1 by using equation (1). Does the result agree with what you obtained in part (b)?