



# Mechatronics Design – Class#22

*Liwei Lin*

Professor, Dept. of Mechanical Engineering  
Co-Director, Berkeley Sensor and Actuator Center  
The University of California, Berkeley, CA94720  
e-mail: [lwlin@me.berkeley.edu](mailto:lwlin@me.berkeley.edu)  
<http://www.me.berkeley.edu/~lwlin>



# Outline

## ◆ Announcement:

- Exam: November 12 (Wed) in class
- Exam grading & solution: Nov. 17 in class
- Project webpage demo/help: November 19 & 24 – class time

## ◆ Design Review II written report

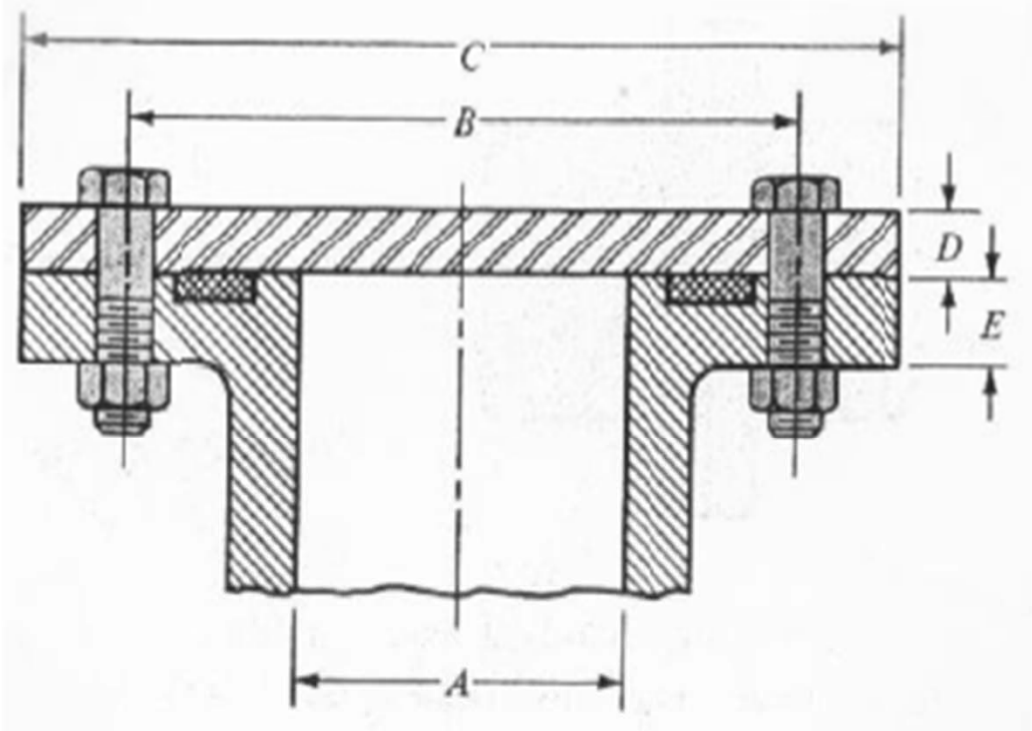
- Due: November 5 in class – exchange with other team
- Due: feedback to other team Nov. 7 (Friday)
- Due: November 10 in class to instructor



### Problem 1 (40%)

The cover of a pressure vessel is held in place by ten  $1/2''$  UNC bolts (proof stress = 85 kpsi). The pressure is 200 Psi and effective area of the cover exposed to the pressure is  $314 \text{ in}^2$ . The ratio of stiffness of the bolt to the connected member is  $1/3$ . Each bolt is tightened to 7500 lb initially, before the pressure is applied.

- Draw a diagram to illustrate safety factors of bolts by showing the important lines in the figure. Please shade the area that is safe to operate and show the definition of safety factor against yielding and safety factor against separation. Derive the equations for safety factors against yielding and against separation, respectively (10%)
- Calculate the safety factor against yielding. (5%)
- Calculate the safety factor against separation. (5%)
- Under what pressure, "separation" will start to occur? (5%)
- Explain in less than 20 words what is happening after the separation? (5%)
- Please calculate the safety factor against yielding (proof stress) right before and right after separation. (10%)

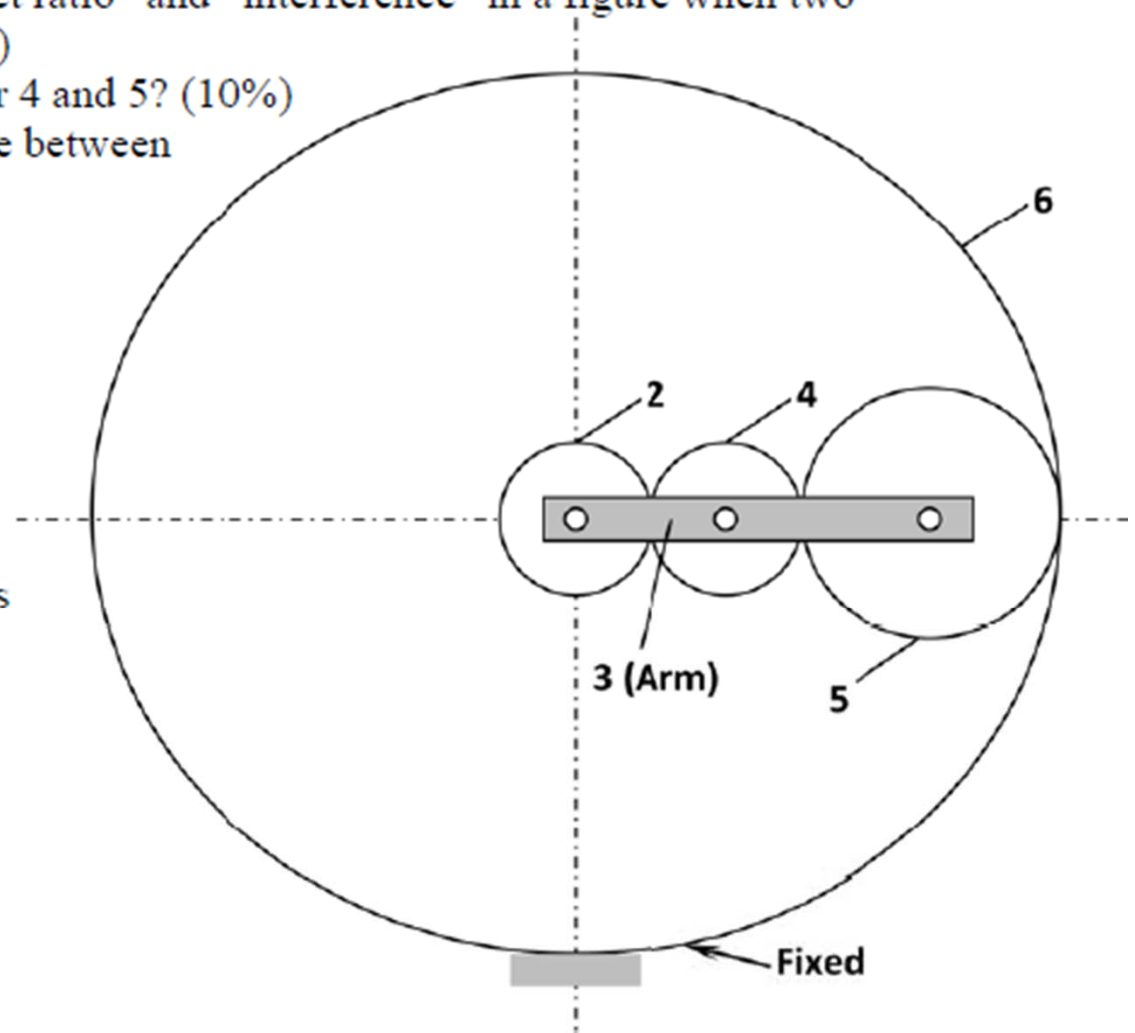




### Problem 2 (60%)

The 24-tooth, 2mm module,  $20^\circ$  pinion shown as element 2 in the figure below rotates clockwise at 1000 rpm and is driven at a power of 2000 Watts. Gears 4, 5, and 6 have 24, 36, and 144 teeth, respectively and the gear efficiency is 100 percent.

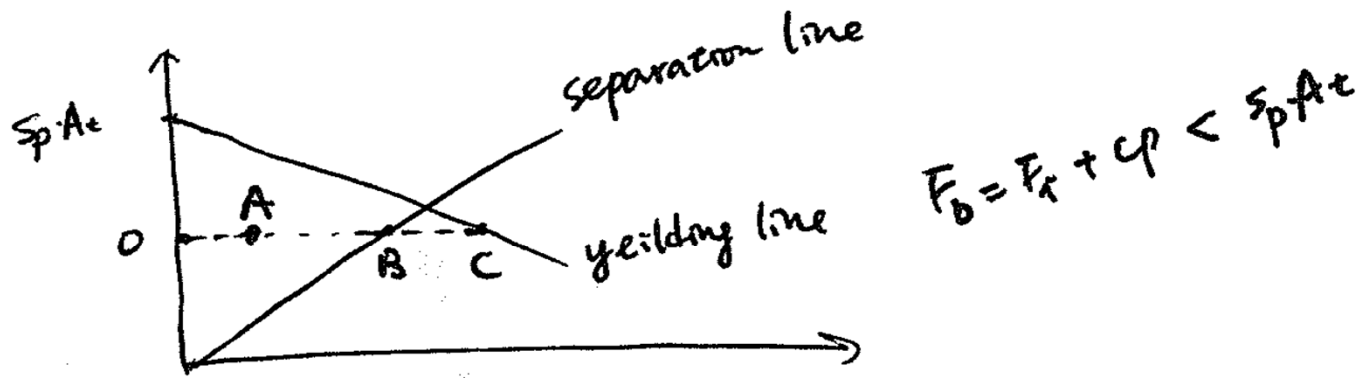
- (a) Illustrate and explain the “contact ratio” and “interference” in a figure when two gears are meshing together. (5%)
- (b) What is the contact ratio for gear 4 and 5? (10%)
- (c) Determine if there is interference between gear 4 and 5? (5%)
- (d) What is the rotational speed and direction of the arm 3? (10%)
- (e) What torque can arm 3 deliver? (10%)
- (f) Draw free-body diagrams of the arm 3 and Gears 2, 4, 5 respectively, and show/analyze/calculate all forces which act upon them in both figures. (20%)





# Problem 1

(a)



$$F_b = F_i + cP < S_p \cdot A \epsilon \quad , \quad F_i + n_y cP = S_p \cdot A \epsilon \Rightarrow n_y = \frac{S_p A \epsilon - F_i}{cP}$$

$$F_m = -F_i + P(1-c) < 0 \quad , \quad F_i = n_s \cdot P(1-c) \Rightarrow n_s = \frac{F_i}{(1-c)P}$$

(b)

$$c = \frac{R_b}{R_b + R_m} = \frac{\frac{1}{3}}{\frac{1}{3} + 1} = 0.25$$

~~$$P_{10} = \frac{1500 \text{ psi} \cdot \pi r^2}{200 \cdot \pi r^2} = \frac{1500 \cdot 3.14 \text{ in}^2}{200 \cdot 3.14} = 117,809 \text{ lb}$$~~

$$P_{10} = \frac{62800}{10} = 6280$$

$$P_i = \frac{62800}{10} = 6280$$

$$n_y = \frac{85000 \cdot 0.142}{(0.25) \cdot 6280} \approx 2.85$$

$$c) \quad n_s = \frac{7500}{(1-0.25) \cdot 6780} \approx 1.59$$

$$d) \quad n_s = 1 = \frac{7500}{(1-0.25)p} \Rightarrow p = 10,000$$

$$\frac{10000 \times 10}{314} = 318 \text{ Psi}$$

e) members are not carrying the external load, only bolt is carry P  
or  $c = 1$

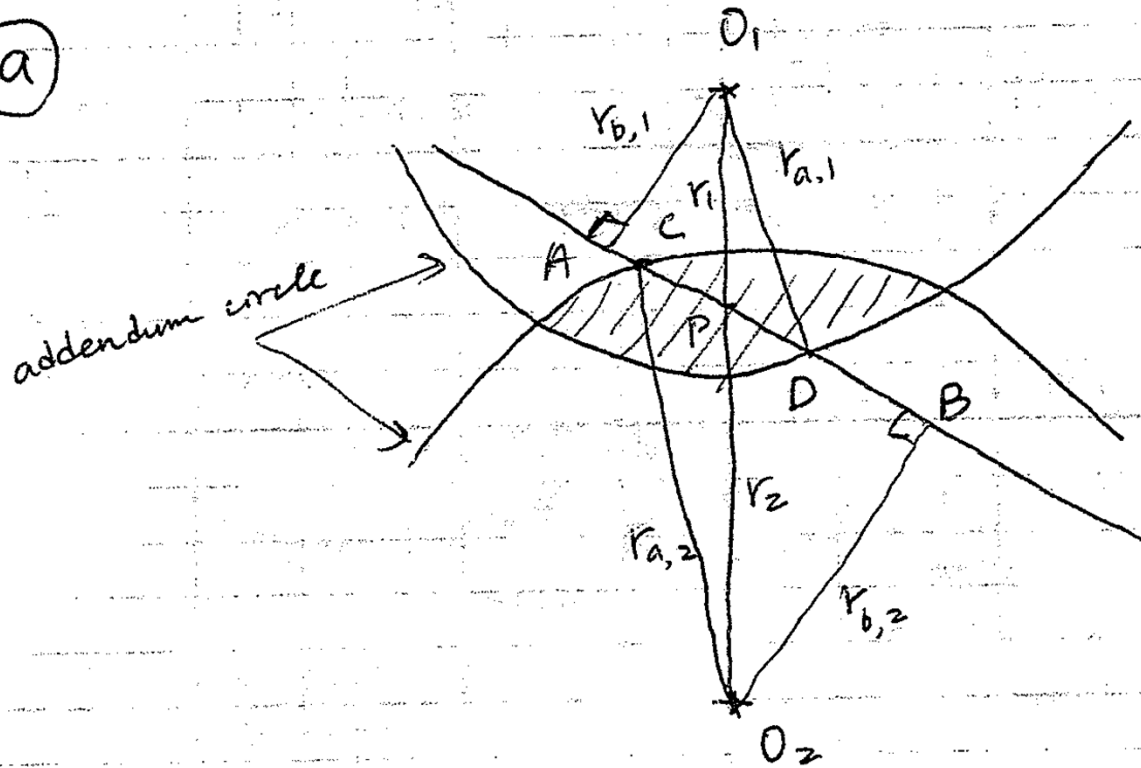
$$f) \quad \text{before} \quad n_y = \frac{S_p \cdot A_c - F_i}{c p} = \frac{85000 \cdot 0.142 - 7500}{0.25 \cdot 10,000} \approx 1.79$$

$$\text{after} \quad F_b = F_i + c p = 7500 + 10,000 = 17500$$

$$S_p \cdot A_c = 85000 \cdot 0.142 = 11985 < 17500 \Rightarrow \text{already yielding}$$

$$\frac{11985}{17500} = 0.68$$

(a)



$r_1, r_2$  - radii of pitch circle  
 $r_{a1}, r_{a2}$  - addendum  
 $r_{b1}, r_{b2}$  - dedendum  
 $O_1, O_2$  - centers of gear 1 & gear 2

contact ratio:  $\frac{\overline{CD}}{2.2 r_b \overline{N}}$

$$\frac{r_b}{N} = \frac{r_{b1}}{N_1} = \frac{r_{b2}}{N_2}$$

$$1.5 < C.R. < 2.0$$

interference if  $\overline{BC} > \overline{AB}$  or  $\overline{AD} > \overline{AB}$



(b)

$$d_1 \text{ (4th gear)} = 2 \cdot 24 = 48 \text{ mm}$$

$$d_2 \text{ (5th gear)} = 2 \cdot 36 = 72 \text{ mm}$$

$$r_{a,1} = r_1 + a = 24 + 2 = 26$$

$$r_{b,1} = r_1 \cos \phi = 24 \cos 20^\circ = 22.55$$

$$r_{a,2} = r_2 + a = 36 + 2 = 38$$

$$r_{b,2} = r_2 \cos \phi = 36 \cos 20^\circ = 33.83$$

$$\overline{CD} = \frac{\sqrt{r_{a,1}^2 - r_{b,1}^2} + \sqrt{r_{a,2}^2 - r_{b,2}^2} - 0.02 \sin \phi}{2\pi \frac{r_{b,1}^2}{N_1}}$$

$$= \frac{13.03 + 17.3 - 20.52}{2\pi \cdot \frac{22.55}{24}} = 1.66$$

(c)

since  $13.03 < 20.52$  OK  $\Rightarrow$  No interference  
 $17.3 < 20.52$  OK





①

$$e = \frac{n_L - n_A}{n_P - n_A} = \frac{0 - n_A}{1000 - n_A} \Rightarrow \text{Aren} = -200 \text{ r/min}$$

$$W_{\%a} = +W_{\%a} \cdot \frac{N_5}{N_6} = -W_{\%a} \cdot \frac{N_4}{N_5} \cdot \frac{N_5}{N_6} = +W_{\%a} \cdot \frac{N_2}{N_4} \cdot \frac{N_6}{N_5} \cdot \frac{N_5}{N_6}$$

$$(0 - W_a) = (1000 - W_a) \cdot \frac{24}{144} \Rightarrow -6W_a = 1000 - W_a$$

$$W_a = -200$$

②

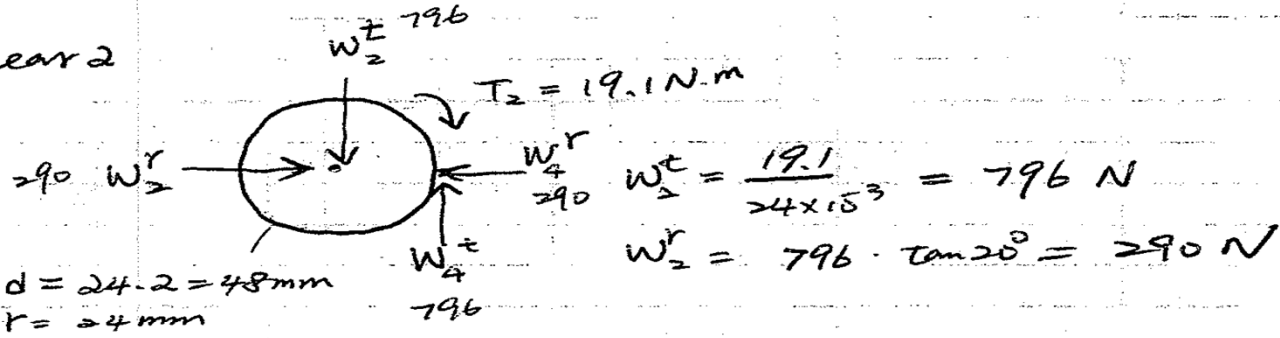
input torque:

$$2000 \text{ W} = T_2 \cdot \frac{2\pi \cdot n \downarrow 1000}{60} \Rightarrow T_2 = 19.1 \text{ N.m}$$

$$1000 \text{ percent efficiency} \Rightarrow 2000 = T_3 \cdot \frac{2\pi \cdot n \downarrow 200}{60} = 95.5 \text{ N.m}$$



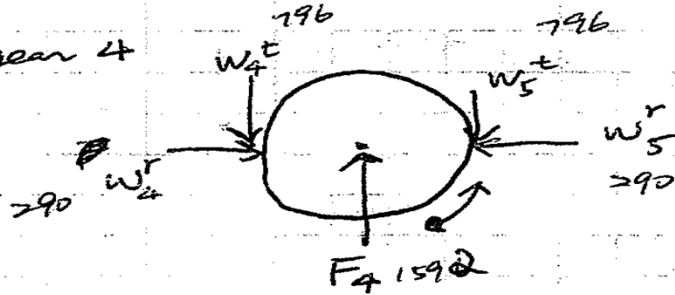
gear 2



$$W_2^t = \frac{19.1}{24 \times 10^{-3}} = 796 \text{ N}$$

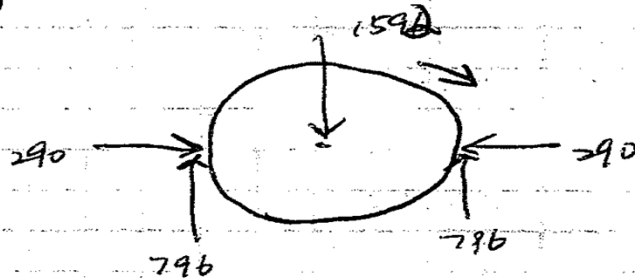
$$W_2^r = 796 \cdot \tan 20^\circ = 290 \text{ N}$$

gear 4

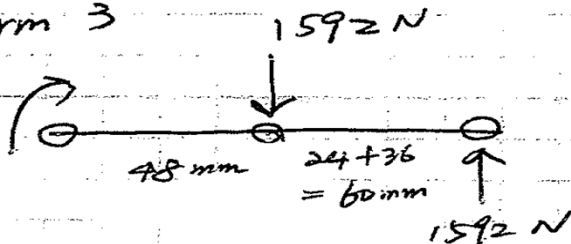


$$F_4 = 2W_4^t = 2 \cdot 796 = 1592 \text{ N}$$

gear 5



Arm 3



$$T_{\text{out}} = T_3 = \frac{1592 \cdot (60 - 48)}{10^{-3}} = \frac{[1592(60 + 48) - 1592 \cdot 48]}{10^{-3}} = 95.5 \text{ N.m}$$