Midterm 1 Solutions : Fall 2006

1) Image #1 (direct image)

$\cdot \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{90cm} + \frac{1}{i} = \frac{1}{-30cm} \Rightarrow i = -22.5cm$

$\Rightarrow \begin{cases} \text{distance from bulb} = 90\,cm - 22.5\,cm = 67.5\,cm \\ m = -\frac{i}{o} = +\frac{1}{4} \ (\Rightarrow \text{upright}) \ \& \ h_i = \frac{h_o}{4} = 1cm \end{cases}$

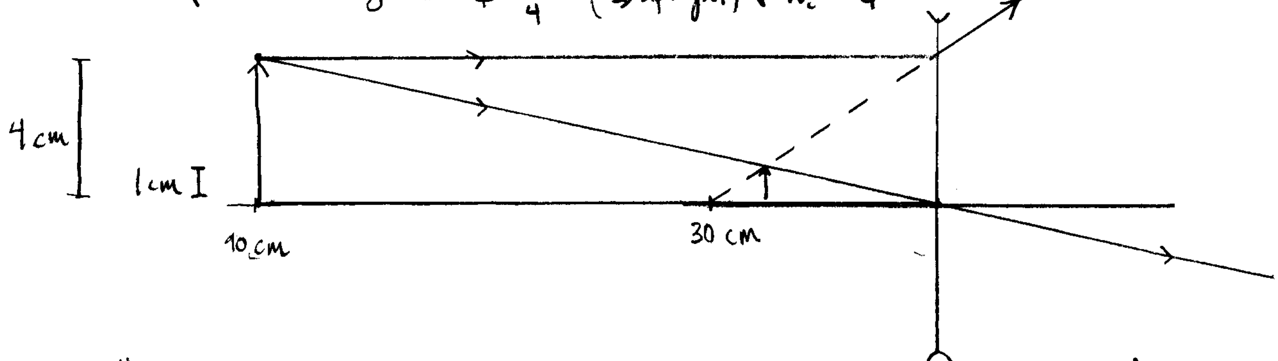
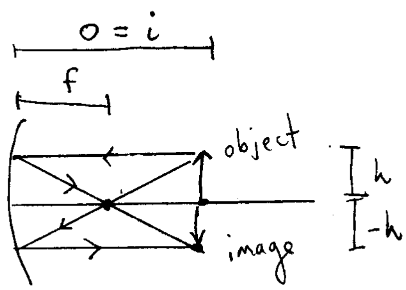


Image #2 (reflected image)



Mirror: The object is at $o = r = 2f$

$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \Rightarrow i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-1}$

$= \left(\frac{1}{f} - \frac{1}{2f}\right)^{-1}$

$= 2f,$

so the image is at the same location as the bulb (the object), but since

$m = -\frac{i}{o} = -\frac{2f}{2f} = -1,$ it is inverted (with same height, 4cm).

Lens: The image of the mirror then acts as an object for the lens, so the ray diagram will be exactly as the one drawn for image #1 above, but inverted.

The result is a final inverted, virtual image located 67.5 cm from the bulb with a height of (negative) 1cm.

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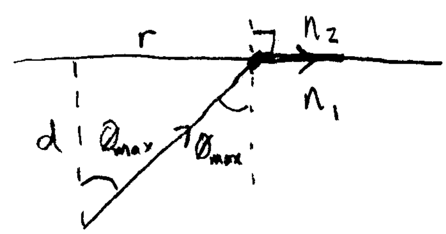
2)



a.) $n_1 \sin\theta_{max} = n_2 \sin 90° \Rightarrow \boxed{\theta_{max} = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

b.) $\sin\theta_{max} = \frac{n_2}{n_1} = \frac{r}{\sqrt{r^2+d^2}} \Rightarrow \boxed{n_2 = \frac{n_1 r}{\sqrt{r^2+d^2}}}$

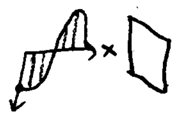
3a) $\langle u \rangle = \langle \epsilon_0 E^2 \rangle = \frac{1}{2}\epsilon_0 E_0^2$

and $\langle u \rangle = \frac{\text{energy}}{\text{volume}} = \frac{\overline{U}_{abs}}{AcT}$

$$\Rightarrow \boxed{E_0 = \left(\frac{2U_{abs}}{\epsilon_0 AcT}\right)^{1/2}}$$

b.) $\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{(I)(4\pi D^2)}{T} = \boxed{\frac{1}{T}\left(\frac{U_{abs}}{A}\right)(4\pi D^2)}$

c.)



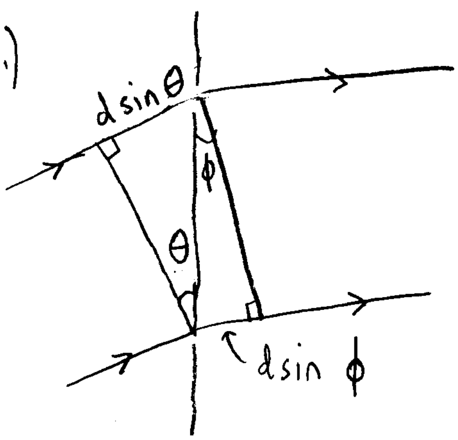
- $\vec{E} \propto \hat{y}$ and $\vec{E}\times\vec{B} \propto \hat{x} \Rightarrow \vec{B} \propto +\hat{z}$
- Since $B_0 = \frac{E_0}{c}$, $\vec{B}_{in} = +\frac{E_0}{c}\sin(kx-\omega t)\,\hat{z}$

d.) No: EM-fields are transverse and this has both propagation direction + $\vec{E}$ proportional to $\hat{x}$.

e.) No: Not a function of $(kx-\omega t)$, so not a wave;
i.e., does not satisfy Maxwell's wave eqn. $\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$

4a.) $\lambda f = c \Rightarrow \lambda = \frac{c}{f} = \frac{2\pi c}{\omega} \Rightarrow k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

b.)



$$\Rightarrow \delta = k\Delta x = k(d\sin\theta - d\sin\phi)$$
$$= \frac{\omega}{c}d(\sin\theta - \sin\phi)$$

c.) $\delta = m\pi$, where $m = \begin{cases}\text{even, maxima}\\ \text{odd, minima}\end{cases} \rightarrow \sin\phi = \sin\theta - \left(\frac{\pi c}{\omega d}\right)m$

(so central max. @ $m = 0 \Rightarrow \phi = \theta$)

e.) $\omega = \omega' \Rightarrow \lambda = \frac{v'}{f'} = \frac{c/n}{\omega/2\pi} = \frac{2\pi c}{n\omega}$ + $k' = \frac{2\pi}{\lambda'} = \frac{n\omega}{c}$

$\rightarrow \sin\phi = \sin\theta - \left(\frac{\pi c}{n\omega d}\right)m$, so central max. still @ $\phi = \theta$ but maxima are closer d.) For light incident normally ($\theta = 0$), we have:

$$I = I_0 \cos^2\left(\frac{\pi d \sin\phi}{\lambda}\right);$$

which has the central maximum at $\sin\phi = 0$.

We now have the cental maximum shifted to $\sin\phi - \sin\theta = 0$, so

$$I \rightarrow I_0 \cos^2\left[\frac{\pi d}{\lambda}(\sin\phi - \sin\theta)\right]$$

is the correct "guess." (This can also be derived algebraically, as in §35-5 of Giancoli.)

- $I = I_0$ when $\frac{\pi d}{\lambda}(\sin\phi - \sin\theta) = m\pi$, $m = \underline{\text{any integer}}$

$$\Rightarrow \sin\phi = \sin\theta + \frac{m\lambda}{d} = \sin\theta + 2m\left(\frac{\pi c}{\omega d}\right)$$ (same as maxima of part c.)

- If one of the slits is covered, there is no interference, so I is constant.

In terms of the I_0 above, $I_{1slit} = \frac{1}{4} I_0$ since the maxima above have

$$I = I_0 = (E_{slit1} + E_{slit2})^2 = 4E_0^2 \quad (\text{where } E_0 \equiv E_{slit1} = E_{slit2})$$

and now we have

$$I = (E_{slit1})^2 = E_0^2 = \boxed{\frac{1}{4} I_0}$$

- If the slits act incoherently, at point P, $I_{incoherent} = \langle I \rangle = \boxed{\frac{1}{2} I_0}$

since it is equally likely to have $I > \frac{1}{2} I_0$ and $I < \frac{1}{2} I_0$ for any random value of $\sin\theta$, and since it is varying rapidly, we just see $I = \frac{1}{2} I_0$ (see Fig. 35-14 of Giancoli).