Berkeley Physics H7B Fall 2014
Dr. Winoto - Final Examination
Thursday, December 18th, 2014

Instruction for the examination (please read carefully):
- In the front of your bluebook, next to your name, please write you SID.
- You may use a calculator without any wireless internet connection and two 8.5"x11" sheet of notes.
- Topic covered: Electricity and Magnetism, Purcell Ch.1-9.
- There are 5 problems (NOT in any order of difficulty, and worth 40 points each), **do them in any order you prefer**.
- Total points for the exam = 200 points for a perfect score
- You have exactly (180-10) minutes to complete the test
- **Show all your work!** Please outline and explain in details all your physical and mathematical reasonings in a clear, rational, step-by-step and logical manner.
- Cross out any parts of your written exam that you would like to discard and not considered as part of your answers.
- In this exam, *c* is always the speed of light, and *e* is always the magnitude of the charge of electron.
- a page of formulas and constants is given in the last page of this exam
- Good luck!

1. (40 points): Helmholtz coils and anti-Helmholtz coils: (see Figure 1):
In one of the homeworks, we showed that at the center of the Helmholtz coils, the magnetic field in the z-direction at the origin O=(0,0,0):
$B_z(0,0,0) = B_o\hat{z}$, for some constant B_o, and that $B_z(0,0,z) = (B_o + \text{const. } z^4)\,\hat{z}$
By using one (or more) of the 4 Maxwell equations (in either integral or differential form) and symmetry:
(a). Please calculate that very near the origin O, $B_x = 0$ and $B_y = 0$.

Also, in the second review session, we showed that at the center of the **anti**-Helmholtz coils, the magnetic field in the z-direction at the origin O=(0,0,0):
$B_z(0,0,0) = 0$, and that for very small z<<a, $B_z(0,0,z)$ is linear in z: $B_z(0,0,z) = \alpha\, z\, \hat{z}$, where α is some constant.
By using one (or more) of the 4 Maxwell equations (in either integral or differential form) and symmetry:
(b). Please calculate, very near the origin O (i.e, for x<<a and y<<a), $B_x(x,0,0)$ and $B_y(0,y,0)$.

2. (40 points): London equation and diamagnetism in superconductivity: (see Figure 2):
Inside a metal superconductor, the current density is given by the London equation:

$$\vec{j} = -\frac{n\,e\text{^}2}{m\,c}\vec{A}\text{ , where }\vec{A}\text{ is the vector potential,}$$

and *e* is the magnitude of the charge of electron, *m* is the mass of electron, and *n* is the number density of the electrons.
(a).
Using the vector identity: $\nabla\times(\nabla\times\vec{F}) = \nabla(\nabla\cdot\vec{F}) - \nabla^2\vec{F}$ and the 4 Maxwell equations, please show that inside the metal superconductor, the B-field obeys the following equation:

$$\nabla^2\vec{B} = \frac{1}{\lambda^2}\vec{B}$$

Please calculate λ.

(b). Consider the following semi-infinite slab of the superconducting metal from part (a) as shown in Figure 2. Outside and at the surface of the superconductor, the B-field is constant everywhere and equal to $B_o\hat{z}$. The B-field deep inside the superconductor is vanishingly zero.
Using part (a), please calculate the B-field anywhere inside the metal as a function of x, for x >=0.

3. (40 points): Electric Field and Magnetic Field of a moving electric-dipole: (see Figure 3):
Consider an electric dipole +e and -e, separated by *a* in the z direction, as shown in Figure 3. Thus, the dipole moment is p = e*a*, pointing in the +z direction. The E-field of the dipole is given by the following expression:

$$\vec{E}(\mathrm{r},\theta) = \frac{p}{4\pi\epsilon_o}\frac{1}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$, where θ is the angle from the z-axis.

The dipole is moving in the +x-direction with constant velocity $v = \frac{4}{5}$c with respect of the lab frame O' (c is the speed of light). At time t' = t = 0, the center of the dipole is at the origin (0,0,0) of the lab frame O'. At this instance in time t' = t = 0 :

(a). Please calculate the E-field $\vec{E}$'(x'=*d*,0,0) and the B-field $\vec{B}$'(x'=*d*,0,0), (and d >> a).

(b). And calculate the E-field $\vec{E}$'(0,0,z'=*d*) and the B-field $\vec{B}$'(0,0,z'=*d*).

4. (40 points): Diamagnetism in a classical hydrogen atom (see Figure 4):
Consider a hydrogen atom with a proton fixed at the origin and an electron in a circular orbit in the x-y plane around the proton with a radius of a_o (given in the table of numerical constant), as shown in Figure 4 (counter-clockwise orbit looking from the top). The circular orbit is held together by the electrostatic interaction between the 2 charges.
(a). Please calculate the speed of the electron v_o in its orbit, and compare it to the speed of light. (please input the numbers only at the end).
(b). Also calculate the period of the circular orbit.
The hydrogen atom is positioned inside and along the z-axis of an infinite solenoid, as shown in Figure 4. The solenoid has a counter-clockwise winding of 5×10^4 turns per meter (looking from the top) and current I(t) given by the following:
I(t<0) = 0 ;
I(0 < t <T) = $I_o\frac{t}{T}$; linearly increasing in time, where I_o= 20 Amp, and T = 1 second.
I(t > T) = I_o;
For the purpose of this problem, and to a good approximation, the radius of the orbit is fixed at a_o.
(c). Please calculate the electric field $\vec{E}$ along the orbit at r=a_o as a function of time, for 0 < t < T.
(d). Calculate the acceleration and the speed of the electron as a function of time, for 0 < t < T.
(e). Finally, please give a numerical answer to the change in speed from v_o at time t=T.

5. (40 points): Displacement current and Induced Magnetic Field
An infinite solenoid, with an axis along the z-axis, has *n* counter-clockwise turns per unit length (looking from the top) and radius *a*. The current flowing through the solenoid has a sinusoidal time dependence: $I(t) = I_o\sin(\omega t)$.
And (you don't need this information to do the problem, but $c\,\frac{2\pi}{\omega}$ >> *a*, meaning that *I(t)* is varying slowly enough).

(a). Please calculate the induced electric-field $\vec{E}$ anywhere inside the solenoid as a function of *r* and *t*.

The Maxwell displacement current density is given by $\vec{j_d} = \epsilon_o \frac{\partial}{\partial t} \vec{E}$

(b). Please calculate $\vec{j_d}$ inside the solenoid as a function of *r* and *t.*

(c). Please calculate the total B-field $\vec{B}$ (original plus induced) anywhere inside the solenoid as a function of *r* and *t.*

Extra credit (please don't feel obliged to do it, truly, unless you have the time):

(d). Please calculate the vector potential $\vec{A}$ anywhere inside the solenoid as a function of *r* and *t.*