Physics 7A Lecture 2 Fall 2014
Midterm 2 Solutions

November 9, 2014

Lecture 2
Midterm 2
Problem 1 Solution

Our general strategy is to use energy conservation, keeping in mind that the force of friction will remove some energy from the system:

$$W_{friction} = \Delta E$$

And we know that the work due to friction is just the force of friction times the distance traveled by the 8kg block. Since the block slides in the direction opposite to the force exerted by friction, the work done by friction is negative.

$$Wfr = -\mu_k m_8 g d$$

All of the initial energy of the system is potential energy. Specifically:

$$E_{initial} = PE_{initial} = m_6 g d$$

Lastly, all of the final energy is kinetic energy. We keep in mind that both masses have the same velocity, since they are connected by a rope.

$$E_{final} = KE_{final} = (½) (m_6 + m_8) v^2$$

So we solve the conservation of energy equation for final velocity to get our answer.

$$-\mu_k m_8 g d = m_6 g d + (½) (m_6 + m_8) v^2$$
$$v = 2.9 \text{ m/s}$$

Physics 7A   Section 2   Midterm 2   Oscar Hallatschek

2a)

First you have to use conservation of Energy to find the velocity of catwoman right before she collides with the bad guy.

$$m_c g h = \frac{1}{2} m_c v_c^2$$

$$v_{c,init.} = \sqrt{2gh}$$

The two people are stuck together after the collision so we can use conservation of momentum to calculate the final velocity of both.

$$m_c v_{c,init.} = (m_c + m_v) v_f$$

$$v_f = \frac{m_c v_{c,init.}}{(m_c + m_v)} = \frac{m_c \sqrt{2gh}}{(m_c + m_v)}$$

2b)

Next we have to find the distance that the two people travel by sliding. You can calculate this two different ways. One way is by relating work and energy the other is by kinematics. I will calculate the answer using kinematics.

The free body diagram for the motion is

The force equation is

$$F_x = -\mu_k N = -\mu_k (m_c + m_v) g = (m_c + m_v) a$$

Hence, the Acceleration is

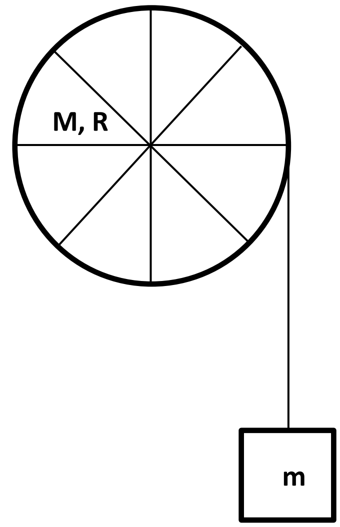
$$a = -\mu_k g$$

Finally to find the distance we use the following kinematics equation

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{-v_i^2}{2a} = \frac{v_i^2}{2\mu_k g} = \frac{{m_c}^2}{(m_c + m_v)^2} \frac{h}{\mu_k}$$

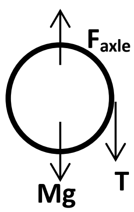
Physics 7A (Section 2)
Problem 3

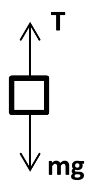


A bicycle wheel consists of a rim of unknown mass M and essentially massless spokes. Suppose we mount the bicycle wheel on a wall, so that the wheel is parallel to the wall, and so that it is free to spin around its center, but not free to move translationally. We wrap a thin, light string around the outer rim of the bicycle wheel and attach a block of mass m to the end of the string, as shown in the figure. In order to figure out the mass M of the rim, an engineering student releases the block from rest and measures the time t that it takes the block to fall through a distance L. How can the student calculate M from knowing L, t, R, m, and g?

Solution *Method 1 – Rotational Dynamics*

<u>*Draw Free Body Diagrams*</u>





<u>*Set Up Equations*</u>

Sum forces for the hanging mass in the y-direction and sum torques about the center for the wheel

$$\sum F = ma = mg - T$$

$$\sum \tau = I_{wheel}\alpha = TR$$

The moment of inertia of the wheel is that same as for a hoop because the spokes are massless.

$$I_{wheel} = MR^2$$

Because the rope is attached to the rim of the wheel the linear acceleration at the edge of the wheel is the same as the acceleration of the block.

$$\alpha = a/R$$

Use linear kinematics to determine the acceleration of the block

$$L = \frac{1}{2}at^2$$

<u>*Solve*</u>

Develop an expression for M as a function of acceleration

$$mg - Ma = ma$$

$$M = m\,(g/a - 1)$$

Calculate acceleration using kinematics

$$a = \frac{2L}{t^2}$$

Solve be plugging in the acceleration into your expression for M

<u>*Solution*</u>

$$M = m\left(\frac{gt^2}{2L} - 1\right)$$

Solution Method 2 - Energy

Set Up Equations

Calculate the initial and final energies of the mass/wheel system. Initially, there is only potential energy from the mass and at the end there is translational kinetic energy from the hanging mass and rotational kinetic energy from the wheel.

$$Initial\ Energy = mgL$$

$$Final\ Energy = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$Conservation\ of\ Energy \rightarrow mgL = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The moment of inertia of the wheel is that same as for a hoop because the spokes are massless.

$$I_{wheel} = MR^2$$

Because the rope is attached to the rim of the wheel the linear velocity at the edge of the wheel is the same as the velocity of the block.

$$\omega = v/R$$

Use linear kinematics to determine the final velocity of the block

$$v^2 = {v_o}^2 + 2aL$$

$$v = v_o + at$$

Solve

Develop an expression for M as a function of final velocity

$$mgL = \frac{1}{2}mv^2 + \frac{1}{2}(MR^2)(\frac{v}{R})^2$$

$$mgL = \frac{1}{2}v^2\ (m + M)$$

$$M = \frac{2mgL}{v^2} - m$$

Calculate final velocity using kinematics

$$v^2 = {v_o}^2 + 2aL = 2aL$$

$$v = v_o + at = at$$

$$a = v/t$$

$$v^2 = 2(\frac{v}{t})L$$

$$v = \frac{2L}{t}$$

Solve be plugging in the final velocity into your expression for M

Solution

$$M = \frac{mgt^2}{2L} - m = m\left(\frac{gt^2}{2L} - 1\right)$$

Problem 4

November 8, 2014

Part A

Figure 1: Two masses being blown apart

We have two balls of different mass forced apart by and explosion. Their total energy is Q. We want to find the fraction of energy that each mass has. By conservation of momentum,

$$m_a v_a + m_b v_b = 0$$

$$v_b = -\frac{m_a}{m_b} v_a$$

By conservation of energy,

$$Q = \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_b v_b^2$$

$$E_a = \frac{1}{2} m_a v_a^2$$

$$E_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2} \frac{m_a^2}{m_b^2} m_b v_a^2$$

Now we will find the fraction of the energy Q that is in each of ball A and B.

$$E_a = \frac{E_a}{Q} Q = \frac{m_a}{m_a + \frac{m_a^2}{m_b}} Q = \frac{m_b}{m_a + m_b} Q$$

$$E_b = Q - E_a = \frac{m_a}{m_a + m_b} Q$$

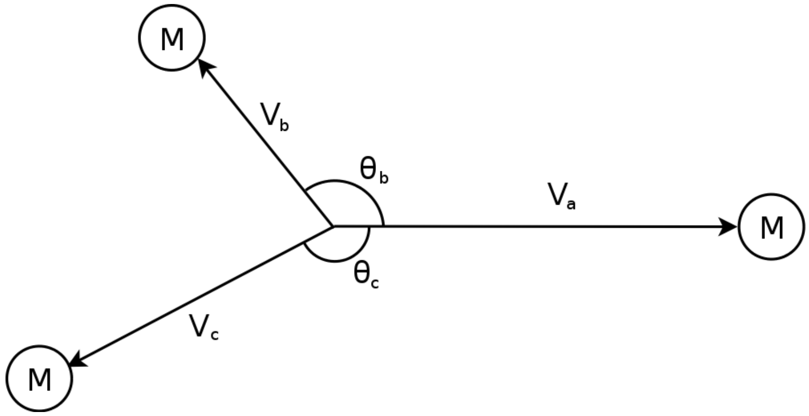


Figure 2: Three masses being blown apart

Part B

Now we have that an explosion blows apart three particles of equal mass. Their total energy is Q. From the conservation of momentum in the x and y directions, we have that

$$mv_b \sin(\theta_b) = mv_c \sin(\theta_c)$$

$$mv_b \cos(\theta_b) + mv_c \cos(\theta_c) + mv_a = 0$$

Reducing these equation, I define v_0 as follows:

$$v_0 = \frac{v_b}{\sin(\theta_c)} = \frac{v_c}{\sin(\theta_b)}$$

$$v_c \cos(\theta_c) + v_b \cos(\theta_b) + v_a = 0$$

$$v_0 \sin(\theta_b) \cos(\theta_c) + v_0 \sin(\theta_c) \cos(\theta_b) = -v_a$$

$$\frac{v_0 \sin(\theta_b) \cos(\theta_c) + v_0 \sin(\theta_c) \cos(\theta_b)}{\sin(\theta_b) \cos(\theta_c) + \cos(\theta_b) \sin(\theta_c)} = \frac{-v_a}{\sin(\theta_b) \cos(\theta_c) + \cos(\theta_b) \sin(\theta_c)}$$

$$v_0 = -\frac{v_a}{\sin(\theta_b) \cos(\theta_c) + \cos(\theta_b) \sin(\theta_c)}$$

Therefore, we can write all velocities in terms of v_0:

$$v_a = -v_0 \left(\sin(\theta_b) \cos(\theta_c) + \cos(\theta_b) \sin(\theta_c)\right) = -v_0 \sin(\theta_b + \theta_c)$$

$$v_b = v_0 \sin(\theta_c)$$

$$v_c = v_0 \sin(\theta_b)$$

The total energy of the particles is

$$Q = \frac{1}{2}mv_a^2 + \frac{1}{2}mv_b^2 + \frac{1}{2}mv_c^2$$

The energy of particle i where $i \in \{a, b, c\}$ is:

$$E_i = \frac{\frac{1}{2}mv_i^2}{Q}Q = \frac{v_i^2}{v_a^2 + v_b^2 + v_c^2}Q$$

The energies of the particles are

$$E_a = \frac{\sin^2(\theta_b + \theta_c)}{\sin^2(\theta_c) + \sin^2(\theta_b) + \sin^2(\theta_b + \theta_c)}Q$$

$$E_b = \frac{\sin^2(\theta_c)}{\sin^2(\theta_c) + \sin^2(\theta_b) + \sin^2(\theta_b + \theta_c)}Q$$

$$E_c = \frac{\sin^2(\theta_b)}{\sin^2(\theta_c) + \sin^2(\theta_b) + \sin^2(\theta_b + \theta_c)}Q$$

Problem 5

Part (a)

The momentum and energy will be conserved since there are no dissipative forces. Also, at the highest point, the block will have the same speed as the ramp and will move only in the horizontal direction with the ramp. Let the speed of the block and the ramp be v_H

Conserving momentum , we have

$$p_i = p_f \tag{1}$$
$$p_i = mv_o \tag{2}$$
$$p_f = mv_H + Mv_H \tag{3}$$

Also, conservation of enery gives

$$E_i = E_f \tag{4}$$
$$E_i = \frac{1}{2}mv_o^2 \tag{5}$$
$$E_f = \frac{1}{2}mv_H^2 + \frac{1}{2}Mv_H^2 + mgH \tag{6}$$

Plug in v_H from above,

$$\frac{1}{2}mv_o^2 = \frac{1}{2}(m+M)(\frac{mv_o}{M+m})^2 + mgH \tag{7}$$

which on simplifying gives

$$v_o = \sqrt{\frac{2(M+m)gH}{M}} \tag{8}$$

Part (b)

Again, since there are no dissipative forces, both energy and momentum will be conserved and the whole climbing up and sliding down can simply be treated as an elastic collision. Let the speed of the block be v_b towards left and the ramp be v_r towars right.

Conserving momentum , we have

$$p_i = p_f \tag{9}$$
$$p_i = mv_o \tag{10}$$
$$p_f = Mv_r - mv_b \tag{11}$$

since the block moves towards the left, it will have negative momentum.

Also, conservation of enery gives

$$E_i = E_f \tag{12}$$
$$E_i = \frac{1}{2}mv_o^2 \tag{13}$$
$$E_f = \frac{1}{2}mv_b^2 + \frac{1}{2}Mv_r^2 \tag{14}$$

Plug in v_b from above,

$$\frac{1}{2}mv_o^2 = \frac{1}{2}Mv_r^2 + \frac{1}{2}m(\frac{Mv_r - mv_o}{m})^2 \tag{15}$$

which on simplifying gives

$$v_r = \frac{2mv_o}{m+M} \tag{16}$$

Part (c)

Let the speed of the block (as seen from the ground) be v_x and v_y in x and y direction. Let the speed of the ramp be v. There are three variables and so we need three equations. Conservation of energy and momentum give us two equations. The third equation is obtained by sitting in the frame of the ramp. In this frame, the block should move up the ramp and hence the direction of its velocity should make an angle of $tan(\theta)$ with the horizontal.

Conserving momentum , we have

$$p_i = p_f \tag{17}$$
$$p_i = mv_o \tag{18}$$
$$p_f = Mv + mv_x \tag{19}$$

Conservation of enery gives

$$E_i = E_f \tag{20}$$
$$E_i = \frac{1}{2}mv_o^2 \tag{21}$$
$$E_f = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}Mv^2 \tag{22}$$

In the frame of the ramp, the block has velocities in x and y direction given by v_{rx} and v_{ry} where

$$v_{rx} = v_x - v \tag{23}$$
$$v_{ry} = v_y \tag{24}$$

Thus,

$$\frac{v_{ry}}{v_{rx}} = \frac{v_y}{v_x - v} = tan(\theta)$$

<u>Alternative method for (c)</u>

Let the two vaiables of interest be speed of the ramp (v_b) and the speed of the block in the frame of the ramp (v_b). Then, in the ground frame, the speed of the block in x and y direction are v_x and v_y given by

$$v_x = v_r + v_b cos(\theta) \tag{25}$$
$$v_y = v_b sin(\theta) \tag{26}$$

Then, conserving energy and momentum gives us

Conserving momentum , we have

$$p_i = p_f \tag{27}$$
$$p_i = mv_o \tag{28}$$
$$p_f = Mv_r + mv_r + mv_b cos(\theta) \tag{29}$$

Conservation of enery gives

$$E_i = E_f \tag{30}$$
$$E_i = \frac{1}{2} mv_o^2 \tag{31}$$
$$E_f = \frac{1}{2} m(v_r + v_b cos(\theta))^2 + \frac{1}{2} m(v_b sin(\theta))^2 + \frac{1}{2} Mv_r^2 \tag{32}$$