Physics 7A Lecture 2 Fall 2014
Midterm 1 Solutions

October 5, 2014

In[60]:= **Solution to Lecture2 Midterm1 Problem1;**

In[67]:= **Part A;**

$V_{x0} = V_0 \, Cos[\theta] = 7 \, Cos[40°]$
$V_{y0} = -V_0 \, Sin[\theta] = -7 \, Cos[40°]$

"Solving for t with the y equation of motion:";

$$y = y_0 + V_{y0} \, t + \frac{1}{2} a \, t^2$$

$$0 = 14 + V_{y0} \, t - \frac{1}{2} g \, t^2$$

$$0 = t^2 - \frac{2 V_{y0}}{g} t - \frac{2}{g} y_0$$

$$t = \frac{\frac{2 V_{y0}}{g} \pm Sqrt\left[\frac{4 V_{y0}^2}{g^2} - 4 \left(-\frac{2}{g} y_0\right)\right]}{2}$$

$$t = \frac{V_{y0}}{g} + Sqrt\left[\frac{V_{y0}^2}{g^2} + \frac{2}{g} y_0\right] = 1.292 \text{ Seconds}$$

"(We dropped the negative time solution, keeping only the positive one)";

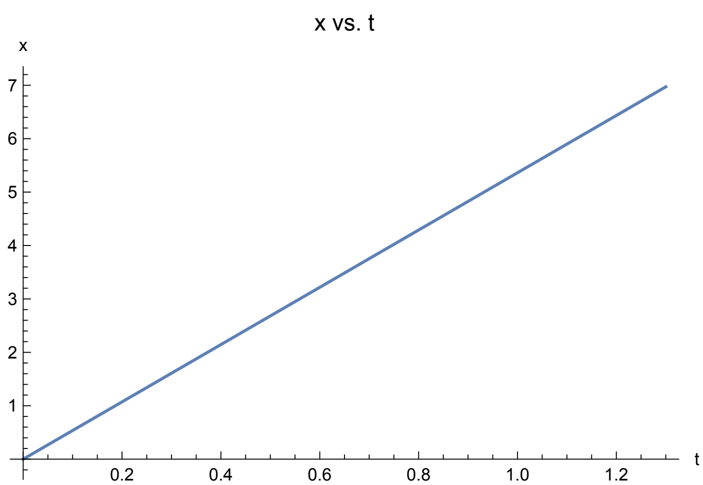
"Plug that t into the x equation of
motion to find the distance traveled in that time";

$x = x_0 + V_{x0} \, t$
$x = 7 \, Cos[40°] * 1.292 = 6.93 \text{ m}$

Part B;

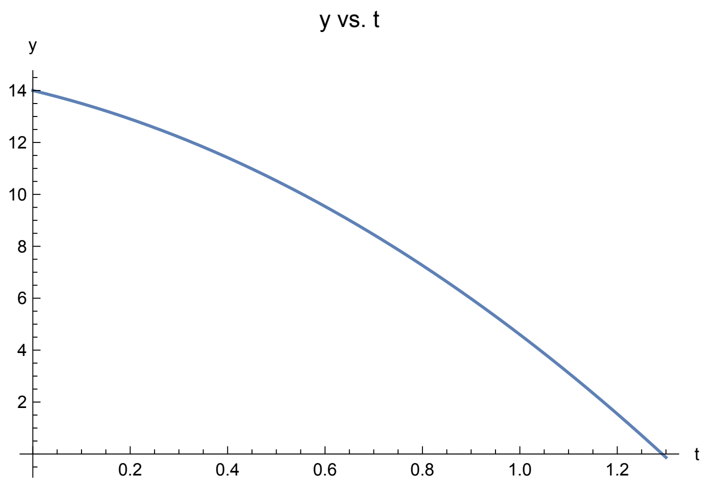
In[8]:= `Plot[7 Cos[40 Degree] t, {t, 0, 1.3}, PlotLabel → "x vs. t", AxesLabel → {"t", "x"}]`

Out[8]=



In[9]:=
```
Plot[14 - 7 Sin[40 Degree] t - 1/2 (9.8) t^2,
  {t, 0, 1.3}, PlotLabel → "y vs. t", AxesLabel → {"t", "y"}]
```

Out[9]=



In[12]:=
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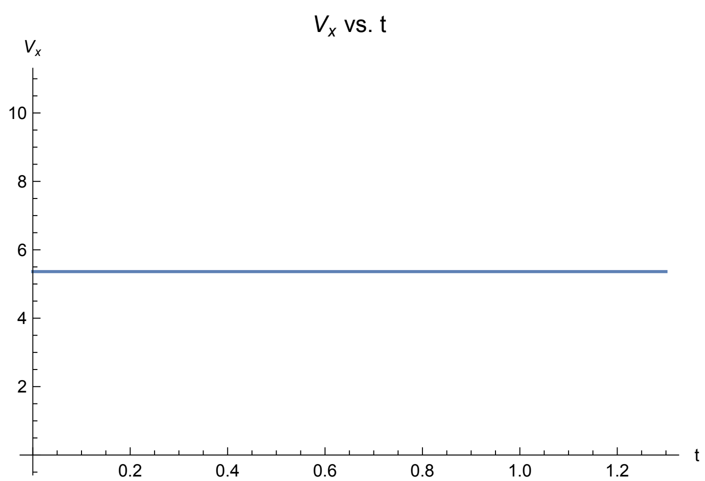
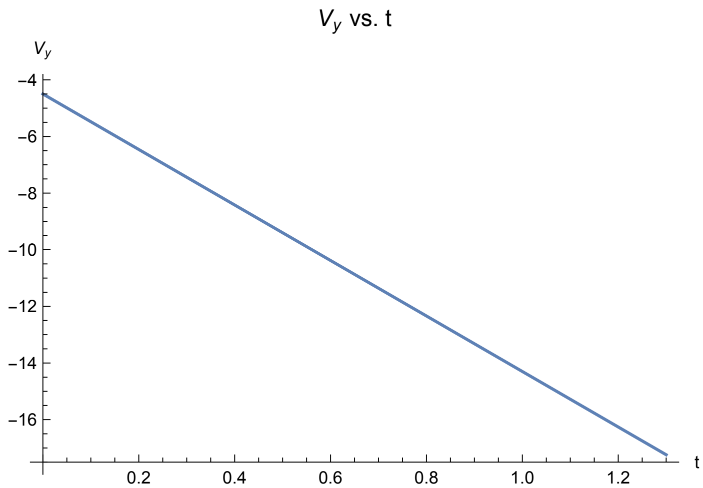
Plot[7 Cos[40 Degree], {t, 0, 1.3}, PlotLabel → "Vx vs. t", AxesLabel → {"t", "Vx"}]
```

Out[12]=



In[13]:=
```
Plot[-7 Sin[40 Degree] - (9.8) t, {t, 0, 1.3},
 PlotLabel → "Vy vs. t", AxesLabel → {"t", "Vy"}]
```

Out[13]=



Part C;

"We calculate the height of the ball when it has moved 4m in the x-direction. If this height is above 1.9m, it has not hit him yet.";

$x = V_{x0}\, t$
$4 = 7 \cos[40°]\, t$
$t = 0.746\, s$

$$y = y_0 + V_{y0}\, t + \frac{1}{2} a t^2$$

$$y = 14 - 7 \sin[40°]\,(0.746) - \frac{1}{2}(9.8)(0.746)^2$$

$y = 7.92\, m$
"Since 7.92m > 1.9m, the man will not be hit";

"Alternative solution: Find the time when the y-position of the ball equals 1.9m, and plug that time into the x equation of motion to find what x the ball is at when y=1.9. If we do it this way, however, we must be careful, since if the snowball reached 1.9m high at around x = 3.5, then it is possible the ball will still hit the man's body. Fortunately, this solution finds t(y=1.9m) = 1.178s, and x(t=1.178s) = 6.317m, which is well after the man, and again we can safely say he won't be hit";

Problem 2 Solution

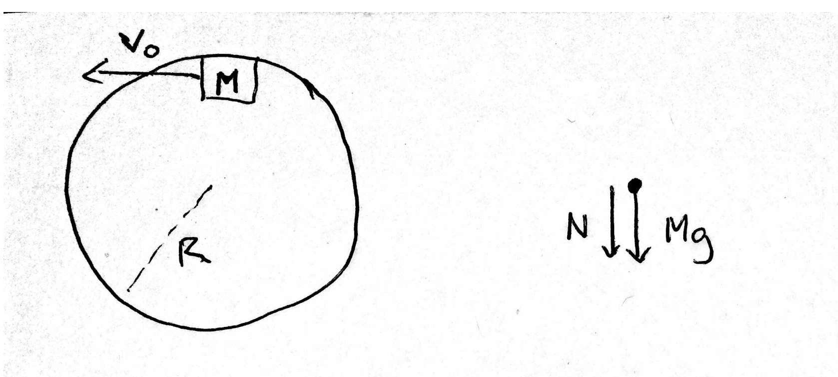
Physics 7A Section 2 Midterm 1 (Hallatschek)

October 2014

Since the person and the motorcycle move together for this problem as a unit, we'll define $M = m_m + m_p$ for the rest of the problem.

Some people solved the problem by considering m_m and m_p separately; this is also correct and will lead to the same answer (assuming both m_m and m_p are executing circular motion with radius R).

(a) At this instant the situation is as shown in diagram below.



The person+motorcycle mass M is executing circular motion of radius R. The FBD shows the vertical forces acting on the mass M. (The problem doesn't mention friction, but friction would point in a horizontal direction and would not affect the answer.) Taking the downward direction as positive and using the circular motion constraint of $a_{central} = \frac{v^2}{R}$ we have

$$N + Mg = M\frac{v^2}{R}. \tag{1}$$

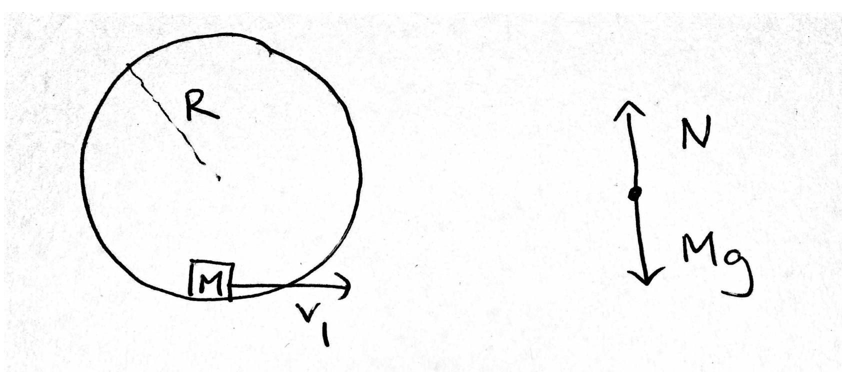
We want to find the minimum speed v_0 that satisfies the above equation; this means we need to minimize the left-hand side of above equation, so we set $N = 0$. Then we have

$$Mg = M\frac{v_0^2}{R},$$

which reduces to

$$v_0 = \sqrt{Rg}. \tag{2}$$

(b) At this instant the situation is as shown in diagram below.



Here the FBD has the normal force pointing upward. (As in part (a), a horizontal force of friction as this instant would not change the reasoning given here.) Again making use of circular motion constraint $a_{central} = \frac{v^2}{R}$ and taking upward direction to be positive, our 2^{nd} Law equation is

$$N - Mg = M\frac{v_1^2}{R}. \tag{3}$$

Solving for the normal force N then gives

$$N = M\left(g + \frac{v_1^2}{R}\right). \tag{4}$$

Note on solutions: A number of students attempted something like fictitious forces to solve this problem (where the problem is implicitly done in a rotating reference frame, and there is a centrifugal force outward). If the student solved the problem correctly with this approach, they were given the benefit of the doubt and awarded full credit. If they solved it incorrectly this way (typically with a sign error from not understanding the direction of the centrifugal force), they were punished more harshly than they would have been for a simple sign error on a straightforward F = ma approach.

Physics 7A

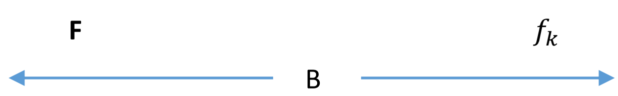
Section 2

Fall Midterm I

Problem 3 Solution

Part A:

The Free Body Diagram for Block B is



The force Equation for part A is

$$F - f_k = m_b a_b$$

Block B is moving with a constant speed and thus has no acceleration. Hence we see that

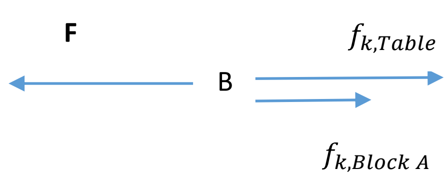
$$F = f_k = \mu_k N$$

Since Block A simply rests on top of Block B, there is no friction of any kind exerted on Block B from A. We can add the masses to get the friction from the interaction of the table and Block B. The Friction is equal to the force.

$$F = f_k = \mu_k N = \mu_k (m_a + m_b) g$$

Part B:

The free body Diagram for Block B is



The force Equation for Part B is

$$F - \mu_k N_a - \mu_k N_b = m_b a_b$$

Once again the Acceleration is zero because Block B is moving at a constant speed.

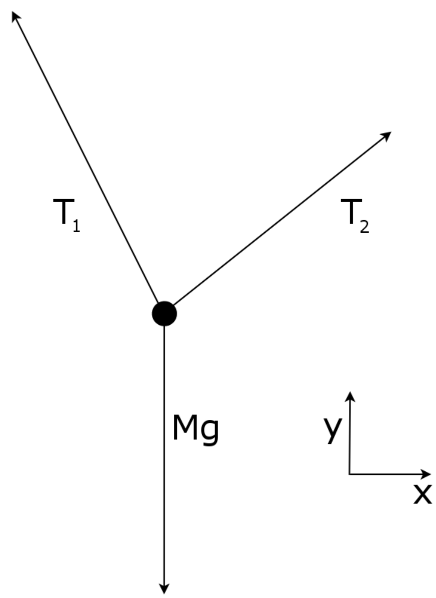
$$F = \mu_k N_a + \mu_k N_b = \mu_k m_a g + \mu_k (m_a + m_b) g$$

Combining terms we get

$$F = \mu_k(2m_a + m_b)g$$

It is good to understand what this problem is saying conceptually. For Block B to have constant speed, in both of the cases, the frictional force must be equal and opposite to the force, F, exerted on Block B.

Problem 4



a) Since the block is not accelerating, the sum of forces in the x direction must be 0. Therefore, the x components of T_1 and T_2 must have the same magnitude ($T_{1x} = T_{2x}$). Knowing that $T_{1y}/T_{1x} = tan(60°)$ is greater than $T_{2y}/T_{2x} = tan(40°)$, we determine that $T_1 > T_2$.

b) Rope 1 will have always be under a larger lension, so Rope 1 will break first. The x and y force equations are:

$$\sum F_x = -T_1 cos(60°) + T_2 cos(40°) = 0$$

$$\sum F_y = T_1 sin(60°) + T_2 sin(40°) - mg = 0$$

Isolating T_2 in the first equation and plugging it in for the second equation, we get:

$$T_2 = T_1 \frac{cos(60°)}{cos(40°)}$$

$$T_1 sin(60°) + T_1 \frac{cos(60°)}{cos(40°)} sin(40°) = mg$$

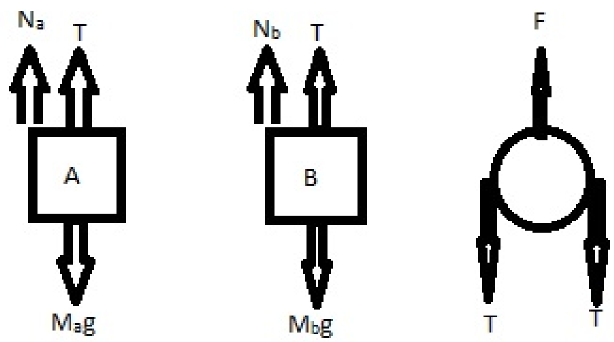
$$mg = T_1 \left(sin(60°) + cos(60°) tan(40°) \right) = 1.286 T_1$$

Plugging in the maximum tension, $T_1 = 5000N$, we get the maximum weight the system can support:

$$mg = 6.43\, kN$$

Problem 5

(a) The most general form of the free body diagrams is attached below. The value of the normal reaction will depend on the magnitude of the force.



Free Body Diagrams

(b) Consider the free body diagram of the pulley. Since the pulley is massless, the net force on it should be zero, otherwise it will lead to infinite acceleration. Thus,

$$F - 2T = 0 \tag{1}$$
$$T = F/2 \tag{2}$$

This holds true always.
(c) We need to consider three cases depending on the magnitude of the force.

- **Case 1** - $F < 2m_B g$

  Since $m_A > m_B$ and $F < 2m_B g$, the tension is not sufficient to lift either of the masses. Thus $a_A = a_B = 0$.

- **Case 2** - $2m_B g < F < 2m_A g$

  In this case, the tension is sufficient to lift the mass B but the mass A stays at rest, on the ground. Thus $a_A = 0$. For mass B, considering the free body diagram,

$$T - m_B g = m_B a_B \tag{3}$$
$$F/2 - m_B g = m_B a_B \tag{4}$$
$$a_B = \frac{F}{2m_B} - g \tag{5}$$

- **Case 3** - $2m_A g < F$

  In this case, the tension is sufficient to lift both the masses A & B. Counterintutively, both the masses can be solved independently if one looks at the free body diagrams. For mass A, considering the free body diagram,

$$T - m_A g = m_A a_A \tag{6}$$
$$F/2 - m_A g = m_A a_A \tag{7}$$
$$a_A = \frac{F}{2m_A} - g \tag{8}$$

  Similarly for mass B, considering the free body diagram,

$$T - m_B g = m_B a_B \tag{9}$$
$$F/2 - m_B g = m_B a_B \tag{10}$$
$$a_B = \frac{F}{2m_B} - g \tag{11}$$

Common things missed out:
All the three cases need to be mentioned.
If you have drawn only one free body diagram for all three cases, then you need to specify normal reaction, N, and mention that it depends on the force.
The magnitude of the acceleration for the two blocks will not be the same in the lab frame.
The magnitude of the acceleration for the two blocks will be the same in the pulley frame. But in that case, one needs to add fictitious force in the equations since the pulley is an accelerating frame.