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MATH 53 FINAL EXAM

Please answer each question on a separate page – you can write on the back of the page. Remember to write your name and id number on EVERY page you turn in. Thanks! Good Luck!

Problem 1 (8 pts). Find the volume of the solid enclosed by the graphs of $z = 0$, $z = \frac{3}{4}|y|$ and cylinders $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$.

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Problem 2 (8 pts). Evaluate the following double integral:

$$6\int_0^1\int_y^1 e^{x^2}y^2\,dx\,dy.$$

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Problem 3 (10 pts).

a) Show that the equation

$$e^{2(z-1)} = xy^2z$$

can be solved for z near the solution $x = y = z = 1$. (That means that we can find $z = z(x, y)$ solving the equation and satisfying $z(1, 1) = 1$).

b) Find $\partial_x z(1, 1)$ and $\partial_y z(1, 1)$.

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Problem 4 (10 pts).

a) Define curl$\mathbf{F}$, where $\mathbf{F} = \langle P, Q, R \rangle$.

b) Show that curl$\nabla f = 0$.

c) Suppose that $\mathbf{F}$ is defined on all of $\mathbf{R}^3$ and that curl$\mathbf{F} = 0$. Can you express $\mathbf{F}$ in terms of one scalar function, and if so how?

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Problem 5 (10 pts).

a) State *Green's theorem.*

b) Prove Green's theorem for a rectangular region:

$$D = \{(x, y) \ : \ a \leq x \leq b, \ c \leq y \leq d\}.$$

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Problem 6 (12 pts).

a) Define the divergence of a vector field $\mathbf{F} = \langle P, Q, R \rangle$.

b) Show that $\operatorname{div} \nabla f = \Delta f$, where

$$\Delta f := \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

c) If E is a solid, S its positively oriented boundary, show that

$$\iiint_E (g\Delta f - f\Delta g)\, dV = \iint_S (g\nabla f - f\nabla g) \cdot \mathbf{n}\, dS.$$

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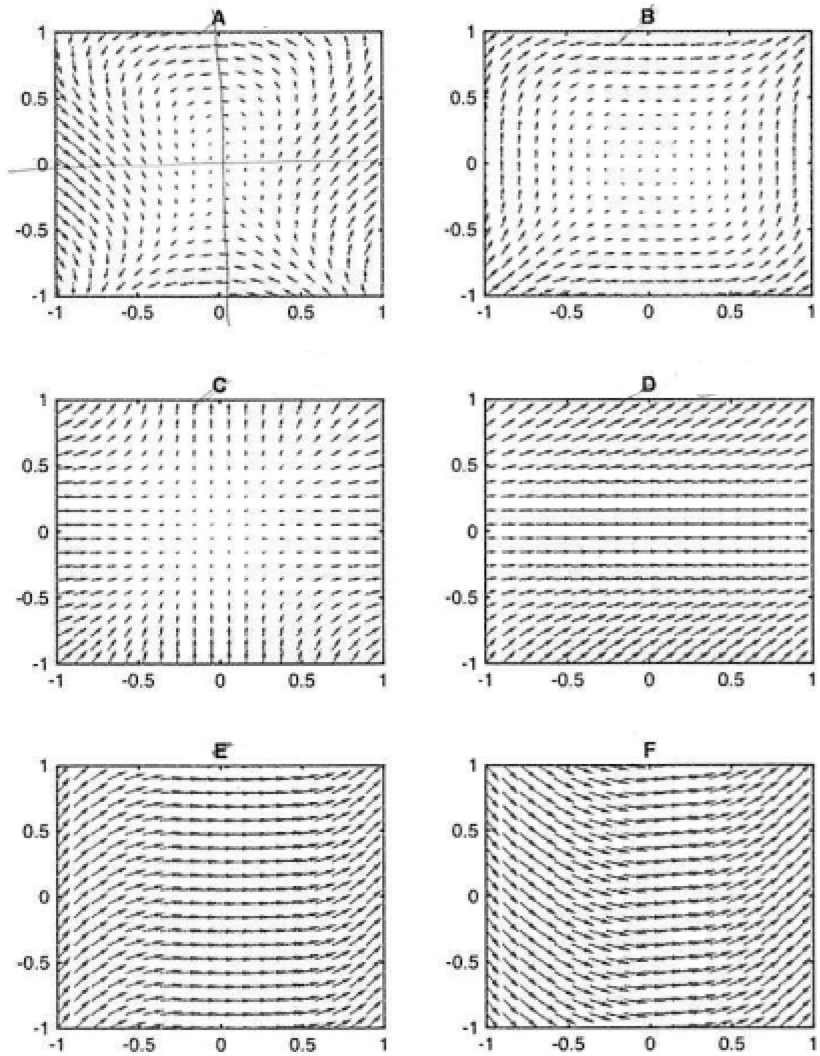
Problem 7 (6 pts). Match the following two dimensional vector fields to the plots below:

$I: \langle \cos x. \sin x \rangle$ $II: \langle x^2 - y^2, x \rangle$ $III: \langle \cos(x^2), \sin(y^2) \rangle$
$IV: \langle y^2, x^2 \rangle$ $V: \langle \cos x, \sin(x^2) \rangle$ $VI: \langle x^2, y^2 \rangle$

Please do not guess: negative points will be given for wrong matches. We have three versions of the exam with different arrangements of answers!



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Problem 8 (12 pts).

a) Parametrize the surface obtained by rotating the curve $x = \cos t$, $z = \sin(2t)$, $y = 0$, $|t| \leq \pi/2$, around the z-axis.

b) Use the divergence theorem to calculate the volume enclosed by the surface in part a). (**Hint:** $\operatorname{div}\langle 0, 0, z\rangle = 1$).

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Problem 8 (12 pts). A cylindrical garbage can is to have volume of $16\pi m^3$ (cubic meters). Find the height h and the radius r of the can which *minimizes* the surface area of the can, including the lid. (**Hint:** this is a constrained minimization problem.)

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Problem 10 (12 pts). Let S be the surface $z = 25 - x^2 - y^2$, $z \geq 16$, oriented so that the unit normal at $(0, 0, 25)$ is $\langle 0, 0, -1 \rangle$. Compute

$$\iint_S \text{curl}\mathbf{F} \cdot \mathbf{n} dS$$

for

$$\mathbf{F} = \left\langle \frac{1}{48}yz, \frac{1}{27}x^2y, \cos(xyz)e^z \right\rangle.$$

(**Hint:** Use Stokes's Theorem.)