Physics 112, Lecture 2
Speliotopoulos
First Midterm, Spring 2015
Berkeley, CA

Rules: *This final exam is closed book and closed notes. In particular, calculators are not allowed during this exam. Cell phones must be turned off during the exam, and placed in your backpacks, or bags.* ***They cannot be on your person.***

Each problem is worth 20 points. We will give partial credit on this final*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

Name: ________________________________ *Disc Sec Number:* ________________________

Signature: _____________________________ *Disc Sec GSI:* ___________________________

Student ID Number: _____________________

1	
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3	
Total	

1. Consider a system in contact with a reservoir with temperature, τ.
   a. Starting with the definition of thermal averages, show that the following is true:
   $$\langle \varepsilon_s^2 \rangle = \tau^2 \frac{\partial}{\partial \tau}\left(\tau^2 \frac{\partial \log Z}{\partial \tau}\right) + \langle \varepsilon_s \rangle^2,$$
   where ε_s is the energy of the system in a state, s, and Z is the partition function.

   b. We define the uncertainty in the average energy by $\Delta\langle \varepsilon_s \rangle \equiv \sqrt{\langle \varepsilon_s^2 \rangle - \langle \varepsilon_s \rangle^2}$. For the system with N particles in a box, what is $\Delta\langle \varepsilon_s \rangle / \langle \varepsilon_s \rangle$? (You will get credit for this part by using the results of part a even if you did not get part a.)

2. Consider a system of N spin-1 particles in contact with a reservoir at temperature, τ, and in the presence of an external magnetic field, B. The states of the a single particle in the system are the following

$$\varepsilon_s = \begin{cases} \mu B & \text{for s=3} \\ 0 & \text{for s=2} \\ -\mu B & \text{for s=1} \end{cases} \quad \text{and} \quad \mu_s = \begin{cases} -\mu & \text{for s=3} \\ 0 & \text{for s=2} \\ \mu & \text{for s=1} \end{cases}.$$

The particles do not interact with one another.

a. What is the average energy, U, for the system?

b. What is U in the limit $\tau \gg \varepsilon$ and in the limit $\tau \ll \varepsilon$?

c. What is the average moment,

$$M = \langle \sum_{n}^{N} \mu_{s_n} \rangle,$$

for the system?

d. Finally, what is the susceptibility, $\chi = dM/dB$, of the system?

3. Figure A on the right shows a system of N_1 *particles in a box* with initial energy, U_{01} and initial multiplicity function $g_1(N_1, U_{01})$, while Figure B shows a system of N_2 particles with initial energy, U_{02} and $g_2(N_2, U_{02})$. The two systems are initially in thermal equilibrium (but not with each other). They are brought into thermal contact with each other in figure C.

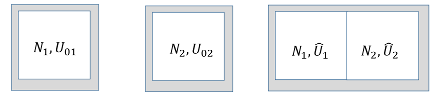

Figure A Figure B Figure C

a. For $U = U_1 + U_2$ and $N = N_1 + N_2$ the product $g_1(N_1, n_1)g_2(N_2, n - n_1)$ has the form,
$$g_1(N_1, U_1)g_2(N_2, U - U_1) = e^{f(N,U,U_1)}.$$
What is $f(N, n, n_1)$? Remember that $g(N, U) = e^{\sigma(N,U)}$in general, and *you can use the entropy of N particles in a box for a system in thermal equilibrium.*

b. When the first system is in the state with $U_1 = \hat{U}_1$, the combined system is in the most probable configuration, and the multiplicity function of the combined system, $g(N, U)$, is sharply peaked at this state. Show that
$$\frac{\hat{U}_1}{N_1} = \frac{\hat{U}_2}{N_2} = \frac{U}{N}.$$

First Midterm
Equation Sheet

Multiplicity functions:

$g(N,s) = \left(\frac{2}{\pi N}\right)^{\frac{1}{2}} 2^N e^{-2s^2/N}$ (dipole system)

$g(N,s) = 3^N \left(\frac{3\sqrt{3}}{2\pi N}\right) e^{-\frac{3u^2}{N}-\frac{3v^2}{N}-\frac{3uv}{N}}$ (spin-1 system)

$g(N,n) = \frac{1}{\sqrt{2\pi}} e^{(N+n)\log(N+n)-n\log n-N\log N}$ (harmonic oscillators)

$g(N,U) = \sum_{U_1} g_1(N_1,U_1)g_2(N_2,U-U_1)$

Probabilities and Averages:

$P(\varepsilon_s) = \frac{e^{-\varepsilon_s/\tau}}{Z} = e^{(\mathcal{F}-\mathcal{E}_s)/\tau}$

$\langle \mathbb{Q}_s \rangle = \frac{1}{Z}\sum_s \mathbb{Q}_s e^{-\varepsilon_s/\tau}$

$U = \langle \varepsilon\_s \rangle = \tau\hat{}2\ \ (\partial \log Z)/\partial\tau$

Entropy:

$\sigma(N,U,V) = \log g(N,U,V)$

$dU = \tau d\sigma - pdV$

$\sigma = -\left(\frac{\partial \mathcal{F}}{\partial \tau}\right)_V$

Free Energy:

$\mathcal{F} = U - \tau\sigma = -\tau \log Z$

Thermodynamic Identities:

$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N$

$C_V = \left(\frac{\partial U}{\partial \tau}\right)_V$

$p = -\left(\frac{\partial U}{\partial V}\right)_\sigma$

$p = \tau\left(\frac{\partial \sigma}{\partial V}\right)_U$

$p = -\left(\frac{\partial \mathcal{F}}{\partial V}\right)_\tau$

$\left(\frac{\partial\sigma}{\partial V}\right)_\tau = \left(\frac{\partial p}{\partial \tau}\right)_V$

For N Non-interacting, non-indentical particles:

$Z = (Z_1)^N$

Math Equations and Approximations:

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\tanh x = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$

$\int_0^\infty e^{-v^2} dv = \frac{\sqrt{\pi}}{2}$

$N! \approx -N + N\log N$

$(1+x)^p \approx 1 + px$

$e^x \approx 1 + x$

$\sinh x \cong x$

$\cosh x \cong 1 + \frac{1}{2}x^2$

N −particles in Box:

$Z = \frac{(Z_1)^N}{N!}$

$\mathcal{F} = -\tau N\left[\log\left(\frac{n_Q}{n}\right) + 1\right]$

$\sigma = N\left[\log\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right]$

$n_Q = \left(\frac{m\tau}{2\pi\hbar^2}\right)^{3/2}$

$U = \frac{3}{2}N\tau$

$pV = N\tau$