EE 20N: Structure and Interpretation of Signals and Systems
Department of Electrical Engineering and Computer Sciences   MIDTERM 1
UC BERKELEY   17 February 2015

FIRST Name ___*Phil*___ LAST Name ___*Terr*___

Lab Day & Time: ___*24/7*___   SID (All Digits): ___*123456789*___

- **(10 Points)** Print your *official* name (not your e-mail address) and *all* digits of your student ID number legibly, and indicate your lab time, on *every* page.

- This exam should take up to 80 minutes to complete. You will be given at least 80 minutes, up to a maximum of 90 minutes, to work on the exam.

- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheet of handwritten, original notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.

- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.

- Please write neatly and legibly, because *if we can't read it, we can't grade it.*

- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*

- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.

- We hope you do a *fantastic* job on this exam.

MT1.1 (105 Points) The input-output behavior of a discrete-time LTI filter H is given by

$$\forall n \in \mathbb{Z}, \quad y(n) = A \sum_{k=-2}^{2} x(n-k),$$

where x denotes the input, y the output, and $A > 0$ a constant parameter.

(a) (10 Points) Select the strongest true assertion from the list below.

  (i) The filter H must be causal.

  (ii) The filter H could be causal, but does not have to be.

  (iii) The filter H cannot be causal.

  To receive credit, you must provide a succinct, but clear and convincing, explanation.

$$y(n) = A\left[\underline{x(n+2) + x(n+1)} + x(n) + x(n-1) + x(n-2)\right]$$

future values of the input

(b) (15 Points) Determine a reasonably simple expression for, and provide a well-labeled plot of, the filter's impulse response values $h(n)$. Your expression for $h(n)$ should be in the form of a linear combination of shifted impulses.

Let $x(n) = \delta(n) \Rightarrow y(n) = h(n) = A\left[\delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)\right]$

$$h(n) = A \sum_{k=-2}^{2} \delta(n-k)$$

(A)
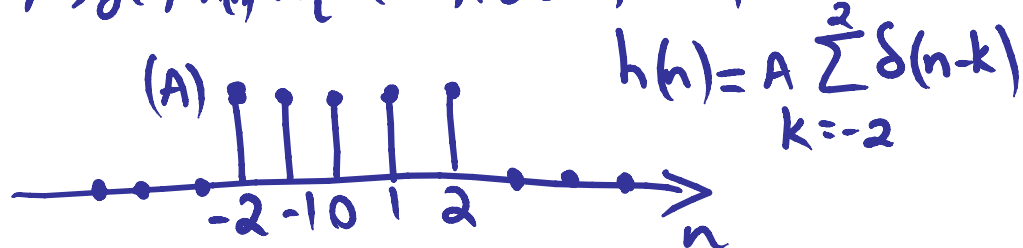
(c) (25 Points) Determine a reasonably simple expression for $H(\omega)$, the frequency response of the filter.

You'll receive full credit if you provide a correct expression that is a *finite sum*, and each term of which is real-valued—that is, no complex term should be seen anywhere in your final expression for $H(\omega)$.

Alternatively, you'll receive full credit if you show that the frequency response can be expressed in the closed form

$$H(\omega) = \alpha \frac{\sin(\beta\omega)}{\sin(\gamma\omega)}$$

for all $\omega \in \mathbb{R}$, *and* if you determine a correct expression or numerical value for each of the scalars α, β, and γ.

Depending on how you tackle this part, you may find it useful to know that

$$\sum_{\ell=M}^{N} \lambda^\ell = \begin{cases} \dfrac{\lambda^{N+1} - \lambda^M}{\lambda - 1} & \text{if } \lambda \neq 1, \\ N - M + 1 & \text{if } \lambda = 1. \end{cases}$$

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n} = A \sum_{n=-2}^{2} e^{-i\omega n}$$

Method I: Write out the five terms and pair them in the following expedient way:

$$H(\omega) = A\left[e^{i\omega 2} + e^{i\omega} + 1 + e^{-i\omega} + e^{-i2\omega} \right] = A\left[1 + (e^{i\omega} + e^{-i\omega}) + (e^{i2\omega} + e^{-i2\omega}) \right]$$

$$H(\omega) = A\left[1 + 2\cos\omega + 2\cos(2\omega) \right]$$

— Using the formula given

Method II: $H(\omega) = A \sum_{n=-2}^{2} (e^{-i\omega})^n = A \dfrac{e^{-i\omega 3} - e^{i\omega 2}}{e^{-i\omega} - 1}$

Now balance the exponents in the numerator and denominator

$$H(\omega) = A \frac{e^{-i5\omega/2} - e^{i5\omega/2}}{e^{-i\omega/2} - e^{i\omega/2}} \cdot \frac{e^{-i\omega/2}}{e^{-i\omega/2}} = A \frac{-2i\sin(5\omega/2)}{-2i\sin(\omega/2)} = A \frac{\sin(5\omega/2)}{\sin(\omega/2)}$$

Note that no matter which method we use, $H(\omega)$ is real-valued. This is not a coincidence. Every real-valued and even $h(n)$ produces a real-valued $H(\omega)$ that is also even in ω.

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(d) (10 Points) Determine A so that if $x(n) = 1$ for all integers n, then $y(n) = 1$ for all integers n.

For this value of A, describe, in plain words, what the filter does to an arbitrary input signal x?

Method I: If $x(n) = 1$ for all n, then $y(n) = A \sum\limits_{n=-2}^{2} 1 = 5A$

To make $y(n) = 1$ for all n, we must have $5A = 1 \Rightarrow A = \frac{1}{5}$

Method II: Recognize that $x(n) = 1 = e^{i0n}$, so the corresponding output is $y(n) = H(0) e^{i0n} = H(0)$. From the expression obtained using Method I, Part (c), we have:

$H(\omega) = A[1 + 2\cos\omega + 2\cos(2\omega)] \Rightarrow H(0) = A(1 + 2 + 2) = 5A$. We must have $H(0) = 1 \Rightarrow A = \frac{1}{5}$.
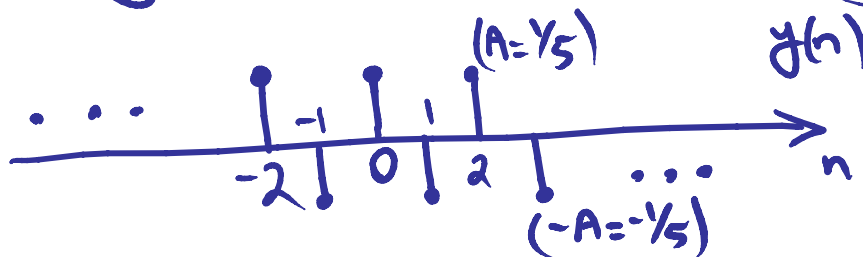
This is a non-causal, five-point moving average filter.

(e) (10 Points) Determine, and provide a well-labeled plot of, the output values $y(n)$, if the input is $x(n) = (-1)^n$ for all integers n. If you're unsure of the specific value that you found for A in part (d), you may leave your expression for, and the plot of, $y(n)$ in terms of A.

$x(n) = (-1)^n = e^{i\pi n} \Rightarrow y(n) = H(\pi) e^{i\pi n} = H(\pi)(-1)^n$

$H(\pi) = A[1 + 2\cos\pi + 2\cos(2\pi)] = A(1 - \cancel{2} + \cancel{2}) = A \Rightarrow$

$y(n) = A(-1)^n$ For the A of Part (d): $y(n) = \frac{1}{5}(-1)^n$

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(f) (20 Points) Determine the simplest expression for, and provide a well-labeled plot of, the output $y(n)$, if the input is

$$\forall n \in \mathbb{Z}, \quad x(n) = \cos\left(\frac{2\pi n}{5}\right) + \left[\sin\left(\frac{2\pi n}{5}\right)\right]^2.$$

If you're unsure of the specific value that you found for A in part (d), you may leave your expression for, and the plot of, $y(n)$ in terms of A.

Depending on how you tackle this part, you may or may not find the following trigonometric identity useful:

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2}.$$

Using the trigonometric identity, we note that

$$\sin^2\left(\frac{2\pi n}{5}\right) = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi n}{5}\right) \implies$$

$$x(n) = \frac{1}{2} + \cos\left(\frac{2\pi n}{5}\right) - \frac{1}{2}\cos\left(\frac{4\pi n}{5}\right)$$

A constant term plus terms involving $e^{\pm i2\pi n/5}$ & $e^{\pm i4\pi n/5}$

Recall that

$$H(\omega) = A\frac{\sin\left(5\omega/2\right)}{\sin\left(\omega/2\right)} \Rightarrow \frac{\sin\left(\frac{5}{2}\cdot\frac{2\pi}{5}\right)}{\sin\left(\frac{1}{2}\cdot\frac{2\pi}{5}\right)} = A\frac{\sin\pi}{\sin\left(\pi/5\right)} = 0$$

Since $H(\omega) = H(-\omega)$, it's also True that $H\left(-\frac{2\pi}{5}\right) = 0$

Furthermore, $H\left(\frac{4\pi}{5}\right) = A\frac{\sin\left(\frac{5}{2}\cdot\frac{4\pi}{5}\right)}{\sin\left(\frac{1}{2}\cdot\frac{4\pi}{5}\right)} = A\frac{\sin(2\pi)}{\sin\left(\frac{2\pi}{5}\right)} = 0 = H\left(-\frac{4\pi}{5}\right)$

Since the input can be expressed as a linear combination of the constant 1 and the complex exponentials $e^{\pm i2\pi n/5}$ and $e^{\pm i4\pi n/5}$, the corresponding response is simply the response To the constant term; all other terms are filtered out.

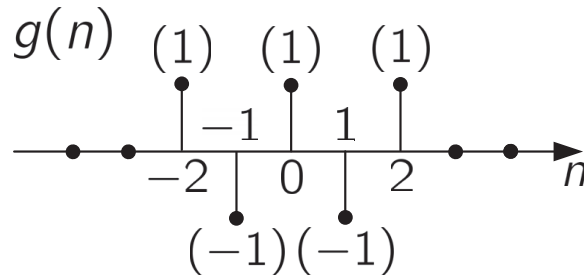The output is $y(n) = \frac{5A}{2}$ ... For $A=\frac{1}{5}$, it is $y(n) = \frac{1}{2}$

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(g) (15 Points) Consider a discrete-time LTI filter G whose impulse response is shown in the figure below.

$$g(n) \quad (1) \quad (1) \quad (1)$$

You may assume that $g(n) = 0$, except where shown explicitly otherwise in the figure.

Determine a reasonably simple expression for $G(\omega)$, the frequency response of the filter G.

Hint: You'll receive full credit if you express $G(\omega)$ in terms of $H(\omega)$ as explicitly and simply as possible. For example, you may express $G(\omega)$ by rewriting—in the simplest form possible—the expression for $H(\omega)$ suggested to you in part (c), even if you did not determine the parameters α, β, and γ correctly. Alternatively, if your expression for $H(\omega)$ in part (c) was a finite sum of real terms, you'll receive full credit here if you express $G(\omega)$ by rewriting that finite sum in the simplest form possible, such that it accounts for the special relationship between $H(\omega)$ and $G(\omega)$.

$$\text{Note } g(n) = (-1)^n h(n) \Rightarrow G(\omega) = \sum_n h(n)(-1)^n e^{-i\omega n}$$

$$\Rightarrow G(\omega) = \sum_n h(n) e^{i\pi n} e^{-i\omega n} = \sum_n h(n) e^{-i(\omega - \pi)} = H(\omega - \pi)$$

$$G(\omega) = H(\omega - \pi) = A \frac{\sin\left[\frac{5}{2}(\omega - \pi)\right]}{\sin\left(\frac{\omega - \pi}{2}\right)} = A \frac{\sin\left(\frac{5\omega}{2} - \frac{5\pi}{2}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{2}\right)}$$

$$= A \frac{\sin\left(\frac{5\omega}{2} - \frac{\pi}{2}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{2}\right)} = A \frac{\cos(5\omega/2)}{\cos(\omega/2)}$$

$$G(\omega) = A \frac{\cos(5\omega/2)}{\cos(\omega/2)}$$