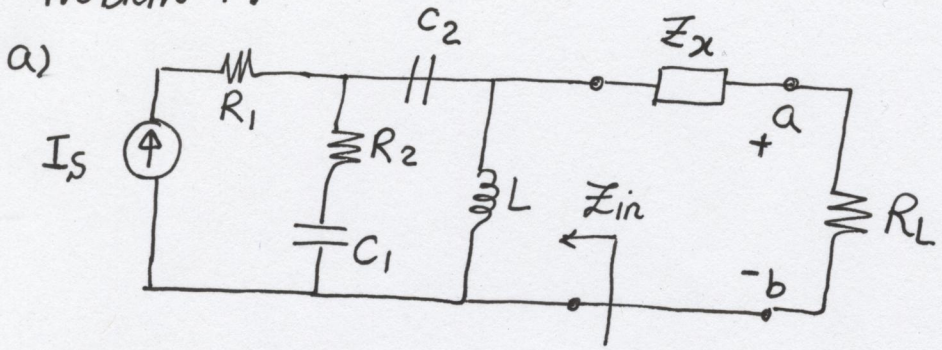
Midterm 2 Solutions:

Problem 1:

a)



$$Z_1 = Z_{C_1} + R_2 + Z_{C_2}$$

$$Z_1 = \frac{1}{j \times 10^6 \times 10^{-6}} + 1 + \frac{1}{j \times 10^6 \times 10^{-6}} = 1 - 2j$$

$$Z_L = j\omega L = j \times 10^6 \times 10^{-6} = j$$

$$Z_{in} = Z_1 \,||\, Z_L = \frac{j(1-2j)}{j+1-2j} = \frac{2+j}{1-j} = \frac{(2+j)(1+j)}{2}$$

$$= \frac{1}{2} + \frac{3}{2}j$$

$$Z_x = -\,Im\{Z_{in}\} = -\frac{3}{2}j$$

b) $$Z_{th} = \frac{1+3j}{2} + Z_x$$

$$Z_x = \frac{-3}{2}j$$

Imag. part Z_{th} should be zero.

Real part of Z_{th} should be minimized.

Problem 2:

a)

$$Z_{i_1} = R_{i_1} \parallel Z_{C_{i_1}} = \frac{R_{i_1} \times \frac{1}{j\omega C_{i_1}}}{R_{i_1} + \frac{1}{j\omega C_{i_1}}} = \frac{R_{i_1}}{1 + j\omega R_{i_1} C_{i_1}}$$

$$\frac{V_1(\omega)}{V_S(\omega)} = \frac{Z_{i_1}}{R_S + Z_{i_1}} = \frac{R_{i_1}}{(R_{i_1} + R_S) + j\omega R_S R_{i_1} C_{i_1}}$$

b) $$Y_{eq} = \frac{1}{R_{O1}} + \frac{1}{R_{i2}} + j\omega(C_{O1} + C_{i2}) = \frac{(R_{i2} + R_{O1}) + j\omega R_{i2} R_{O1}(C_{O1} + C_{i2})}{R_{O1} R_{i2}}$$

$$Z_{eq} = \frac{1}{Y_{eq}}$$

$$V_2 = -g_{m_1} V_1 Z_{eq}$$

$$\frac{V_2}{V_1} = \frac{-g_{m_1} R_{O1} R_{i2}}{(R_{i2} + R_{O_1}) + j\omega R_{i2} R_{O_1}(C_{O_1} + C_{i2})}$$

c) $$Z_{O_2} = R_{O_2} \parallel \frac{1}{j\omega C_{O_2}} = \frac{R_{O2}}{1 + j\omega R_{O_2} C_{O_2}}$$

$$V_{out} = g_{m_2} V_2 Z_{O_2}$$

$$\frac{V_{out}(\omega)}{V_2(\omega)} = \frac{g_{m_2} R_{O2}}{1 + j\omega R_{O_2} C_{O_2}}$$

d) $$\frac{V_{out}(\omega)}{V_S(\omega)} = \frac{V_1(\omega)}{V_S(\omega)} \cdot \frac{V_2(\omega)}{V_1(\omega)} \cdot \frac{V_{out}(\omega)}{V_2(\omega)}$$

$$\frac{V_{out}(\omega)}{V_S(\omega)} = \frac{R_{i_1}}{(R_{i_1} + R_S) + j\omega R_S R_{i_1} C_{i_1}} \cdot \frac{-g_{m_1} R_{O_1} R_{i2}}{R_{i2} + R_{O_1} + j\omega R_{i2} R_{O_1}(C_{O_1} + C_{i2})} \cdot \frac{g_{m_2} R_{O2}}{1 + j\omega R_{O_2} C_{O_2}}$$

Problem 3:

a)

$$\frac{V_{S1}}{R_1} = \frac{-V_{o1}}{R_2} - \frac{V_{out}}{R_3}$$

$$\frac{V_{o1}}{R_5} = \frac{-V_{out}}{R_4} \Rightarrow V_{o1} = -\frac{R_5}{R_4} V_{out}$$

$$\frac{V_{S1}}{R_1} = \frac{R_5}{R_2 R_4} V_{out} - \frac{V_{out}}{R_3}$$

$$\frac{V_{out}}{V_{S1}} = \frac{R_2 R_3 R_4}{R_1 (R_3 R_5 - R_2 R_4)}$$

b) $$V_{out}(o) = \left(\frac{-R_2}{R_1}\right)\left(\frac{-R_4}{R_5}\right) = \frac{R_2 R_4}{R_1 R_5}$$

$$V_{out}(\infty) = 0$$

$$\frac{V_{S1}}{R_1} + \frac{V_{o1}}{R_2} + \frac{C dV_{out}}{dt} = 0$$

$$V_{o1} = -\frac{R_5}{R_4} V_{out} \qquad V_{S1}(t>o) = 0$$

$$\Rightarrow -\frac{R_5}{R_4 R_2} V_{out} + \frac{C dV_{out}}{dt} = 0$$

$$\frac{dV_{out}}{dt} - \frac{R_5}{C R_4 R_2} V_{out} = 0 \Rightarrow \tau = \frac{-R_4 R_2 C}{R_5}$$

$$\Rightarrow V_{out}(t) = \frac{R_2 R_4}{R_1 R_5} e^{\frac{R_5 t}{R_2 R_4 C}}$$

c)

$$\frac{V_{out}(\omega)}{V_{S1}(\omega)} = \frac{R_2 Z_3 R_4}{R_1(Z_3 R_5 - R_2 R_4)} = \frac{1}{1 - 318\pi j}$$

$$V_{S1}(\omega) = 1$$

$$\Rightarrow V_{out}(\omega) = \frac{1}{1-318\pi j} = \frac{1}{\sqrt{1+(318\pi)^2}} \angle \tan^{-1}(318\pi)$$

$$v_{out}(t) = \text{Real}\left\{V_{out} e^{j\omega t}\right\} = \frac{1}{\sqrt{1+(318\pi)^2}} \cos(318\times 10^2 \pi t + \tan^{-1}(318\pi))$$

Problem 4:

$V_C(0) = 0 \quad \Rightarrow V_R(0) = 1^V$

$V_R(\infty) = 0 \qquad \tau = RC$

$V_R(t) = V_R(\infty) + [V_R(0) - V_R(\infty)]e^{-\frac{t}{\tau}}$

$$V_R(t) = \begin{cases} e^{-\frac{t}{\tau}} & t \geq 0 \\ 0 & t < 0 \end{cases}$$