Physics H7C Midterm 2 Solutions

Eric Dodds

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1 Qualitative questions

a) The angular resolution of a space based telescope is limited by the wave properties of light, that is, by diffraction. The relevant parameters are the wavelength λ of the light being viewed and the diameter D of the objective; the shorter the wavelength the smaller the angular separation necessary to resolve sources. That is, the angular resolution should scale like the angles of diffraction fringes. Quantitatively one uses the Rayleigh criterion for point sources separated by an angle θ to be resolved:

$$\theta = 1.220 \frac{\lambda}{D}$$

which with the given numbers gives

$$\theta = 2.684 \times 10^{-7}.$$

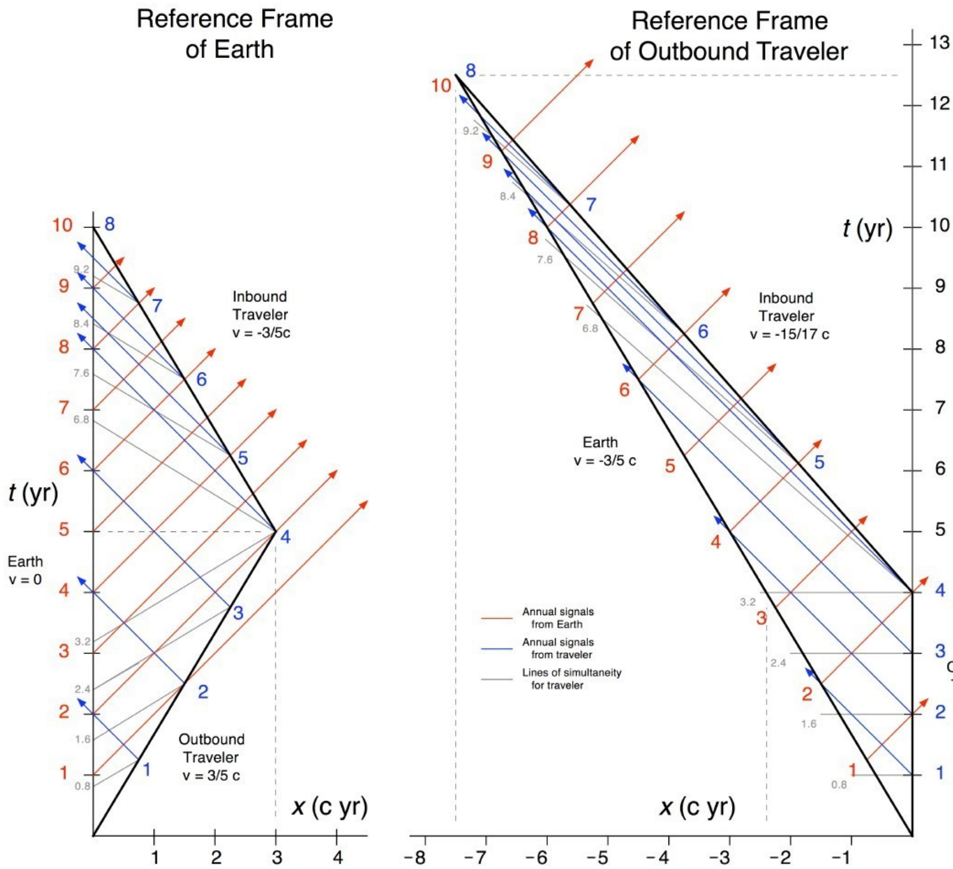
(We weren't looking for you to get the 1.220 exactly; the important thing is how and why the resolution depends on λ and D.)

b) The diffraction pattern is the usual Airy disk pattern we've seen many times: a bright central spot surrounded by dimmer concentric rings. You can argue this in various ways, such as by recalling that the Fourier transform of the even "tophat" illumination is a "sinc" function, or by using the Huygens wavelet model.

Optics is time-reversal symmetric, so you can get the beam pattern by time-reversing outgoing light. The question was slightly unclear, though, so I also accepted answers saying time-reversal doesn't make sense since the telescope isn't looking at the same image that it would project.

c) In the Earth's frame, the "clock" that is the decaying muons appears to run slow (time dilation), as any moving clock must do. So the muons last longer and make it farther. From the muons' perspective, the atmosphere of the earth is quite thin (length contraction) and so they have enough time even in their short lives to traverse it.

d) Here's a picture I stole from someone on the internet:



The asymmetry is that one of the twins is in an inertial reference frame while the twin who travels out and back is not. Remember that not all reference frames are equivalent, only inertial ones. As you can see from the diagram, both twins receive redshifted signals while the traveler is on her way out, but the symmetry is broken as on the way back the traveler receives blueshifted signals while her twin still receives redshifted signals for a time. (Your drawing doesn't have to be that detailed or precise!)

2 Mirror mirror

a) First let's find the travel time Δt in S. The light pulse travels d to the mirror, then $(d - v\Delta t)$ back to the rocket, since the rocket is $v\Delta t$ closer to the mirror by the time the light returns. The light pulse travels at speed c, so

$$c\Delta t = 2d - v\Delta t.$$

The total travel time is thus

$$\Delta t = \frac{2d}{c+v} = \frac{10}{9}\frac{d}{c}. \tag{1}$$

We can indeed use time dilation to find the travel time $\Delta t'$ in the rocket's frame S', because in that frame the events of the emission and absorption of the light pulse happen at the same position, namely the front of the rocket. Time dilation can be derived in several ways; one is by calculating the spacetime interval between the events

$$\begin{aligned}(\Delta s)^2 &= (c\Delta t)^2 - (\Delta x)^2 \qquad\qquad &= (c\Delta t')^2 - \cancel{(\Delta x')^2} \\ &= (c\Delta t)^2 - (v\Delta t)^2 &= (c\Delta t')^2\end{aligned} \tag{2}$$

$$\Rightarrow (\Delta t')^2 = (\Delta t)^2(1-\beta^2)$$

$$\Rightarrow \Delta t' = \frac{\Delta t}{\gamma}. \tag{3}$$

In this case

$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{2d}{(c+v)\gamma} = \frac{2}{3}\frac{d}{c}. \tag{4}$$

This result can also be written in a way that may make it look familiar:

$$\Delta t' = \frac{2d}{c}\frac{\sqrt{1-\beta^2}}{1+\beta} = \sqrt{\frac{1-\beta}{1+\beta}}\frac{2d}{c}. \tag{5}$$

b) The frequency of the light is unaffected by the mirror *if* the mirror is at rest. So let's consider the light pulse's trip in S where we know what happens with the reflection, then go back to the rocket's frame when we need to. In the mirror's frame, the light pulse heading towards the mirror gets blueshifted:

$$\omega = \sqrt{\frac{1+\beta}{1-\beta}}\omega' \tag{6}$$

then bounces off the mirror and goes the other way with the same frequency ω. The rocket receives a light pulse whose source (the mirror) is approaching. So the light pulse appears blueshifted again:

$$\omega_{\text{ref}} = \sqrt{\frac{1+\beta}{1-\beta}}\omega = \frac{1+\beta}{1-\beta}\omega' = 9\omega'. \tag{7}$$

3 Measuring the mass of the pion

We treat the neutrino as exactly massless, so its energy-momentum relation is simply $E = pc$. Conservation of relativistic energy implies

$$m_\pi c^2 = K + m_\mu c^2 + E_\nu \tag{8}$$

where E_ν is the energy of the neutrino. We can determine E_ν using conservation of relativistic momentum. Since the net momentum of the system is zero and there are only two final state particles, these particles must have opposite momenta. The direction of these momenta is the only direction in the problem; conservation of the component of momentum along this direction implies

$$0 = \sqrt{(K + m_\mu c^2)^2/c^2 - m_\mu^2 c^2} - E_\nu/c \tag{9}$$

where I used $E^2 = p^2c^2 + m^2c^4$ to determine the momentum of the muon. Putting these equations together we have

$$\begin{aligned} m_\pi c^2 &= K + m_\mu c^2 + \sqrt{(K + m_\mu c^2)^2 - m_\mu^2 c^4} \\ &= K + m_\mu c^2 + \sqrt{K^2 + 2K m_\mu c^2} \qquad (10) \\ &= 4.6 \text{ MeV} + 106 \text{ MeV} + \sqrt{(4.6 \text{ MeV})^2 + 2 \times 4.6 \text{ MeV} \times 106 \text{ MeV}} \\ &= 142 \text{ MeV} \qquad (11) \end{aligned}$$

which is intermediate between the electron mass and the proton mass[1].

4 Photon-electron scattering

a) A free electron cannot absorb a photon. We can prove this quickly using energy and momentum conservation. Consider the process in the center of momentum frame, where the photon has momentum E/c and the electron has the same momentum in the opposite direction. After the photon is absorbed, only the electron remains and so its momentum must be zero by conservation of momentum. Then the total energy of the system is just the rest energy $m_e c^2$ of the electron. But the initial energy of the system is

$$E_i = m_e c^2 + K_e + E \tag{12}$$

where the kinetic energy K_e of the electron and the photon energy E are positive. So energy is not conserved in this process, meaning it is not kinematically allowed.

[1] The accepted experimental value for the pion mass is a few MeV less than what we found, but that shouldn't be too surprising given the slightly rough numbers we used.

b) The situation changes if there's a proton around. Let's think in the center of momentum frame again. The hydrogen atom (that is, the proton and electron) must be at rest in this reference frame after absorbing the photon. But because the system is composite, determining the total momentum doesn't determine the total energy! The kinetic energy of the system ends up in kinetic energy of the constituents of the atom moving relative to one another and in changing the potential energy of the system.