FINAL EXAM

Instructor: Prof. A. LANZARA

TOTAL POINTS: 165

TOTAL PROBLEMS: 7

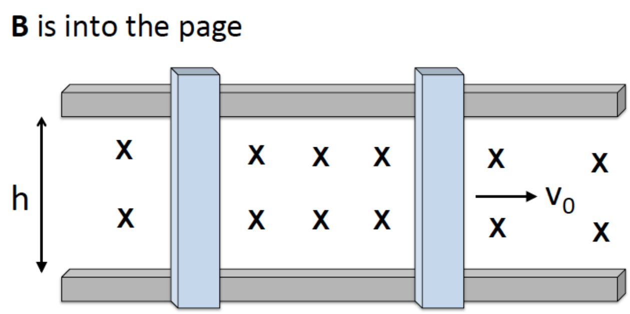
Read the whole exam before starting to solve problems. Start with the ones you are more familiar with to secure points.
Show all work, and take particular care to explain what you are doing. Show a logical progression of steps from equations on the equation sheet to your final answer. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period. All answers should be in terms of variables.

Please use one sheet per problem and have them appear in order in your green book. If you need extra room for a problem, place it at the end, after all of the other problems and include a note in the original problem to guide the grader there.

GOOD LUCK!

PROBLEM 1 (Points 25)

Two conducting rods, each with mass M and resistance R, are placed across long horizontal parallel conducting rails that have negligible resistance. There is a uniform magnetic field B at right angles to the rods and the rails (see diagram below). The conducting rods slide without friction along the rails. The distance between the rails is h.



Suppose the rod on the right is pulled to the right with a constant velocity v_0.

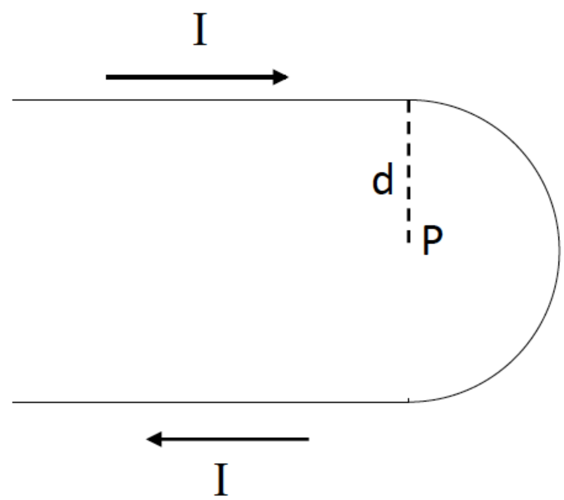
a) (15pts) How does the rod on the left respond? In particular, find the velocity v(t) of the rod on the left as a function of time, assuming that initially its velocity is zero.

b) (10pts) Describe the motion of the two rods after a very long time and give a brief answer why this follows relatively directly from Faraday's Law.

PROBLEM 2 (Points 25)

A long wire is bent into the hairpin-like shape shown in the figure.

a) (6pts) What is the direction of the magnetic field at the indicated point P, which lies at the center of the half-circle?
b) (13pts) What is the magnitude of the magnetic field at that point?
c) (6pts) Suppose an electron is at point P and has a velocity v pointing into the page. What force does the magnetic field exert on it?



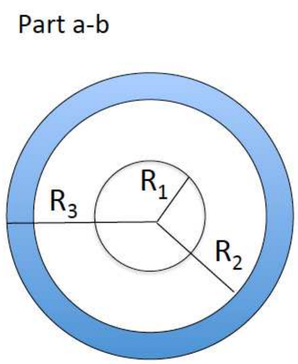
PROBLEM 3 (Points 25)

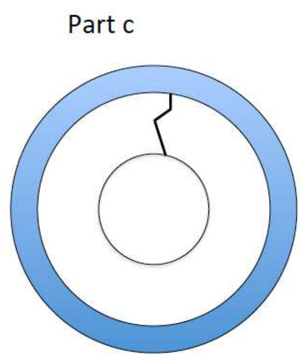
A thin conducting shell of radius R_1 is centered inside a thick conducting shell with inner radius R_2 and outer radius R_3. The inner shell has positive charge $2Q_0$ on it, while the outer shell has net charge $-Q_0$ on it.

a. (3pts) What is the charge residing on the inner surface of the thick shell (at $r=R_2$)? Explain your reasoning.
   (3pts) What is the charge residing on the outer surface of the thick shell (at $r=R_3$)? Explain your reasoning.
   Find:

b) (3pts) The potential at the outer surface of the thick shell $V(r=R_3)$
   (3pts) The potential at the inner surface of the thick shell $V(r=R_2)$
   (3pts) The potential at the surface of the thin shell $V(r=R_1)$
   (3pts) The potential at the center of the thin shell

a. (7pts) The two spheres are now connected by a thin conducting wire. Describe what happens after a long time passes to the charge on the inner and outer shell and to the difference of potential.

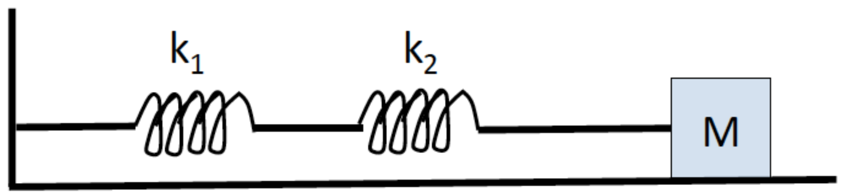




PROBLEM 4 (Points 20)

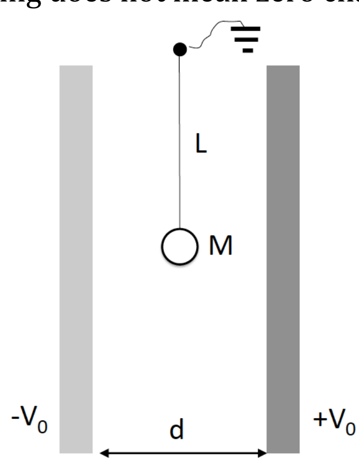
Two springs with spring constants as shown are joined and connected to an object with mass m, the arrangement being free to oscillate on a horizontal frictionless surface as in the figure below.

a) (10pts) Sketch the electromagnetic analog of this mechanical oscillating system.
b) (10pts) Find an expression for the current as a function of time in this equivalent circuit.



PROBLEM 5 (Points 25)

Two large parallel vertical conducting plates separated by a distance d are charged so that their potentials are $+V_0$ and $-V_0$. A small conducting ball of mass M and radius R (R<<d) is hung midway between the plates. The thread of length L supporting the ball is a conducting wire connected to ground, so the potential of the ball is fixed at V=0. The ball hangs straight down in stable equilibrium when V_0 is sufficiently small. Find for which value of V_0 the ball does not hang straight anymore. (Hint: consider the forces on the ball when it is displaced by a distance x<<L. Pay attention: Grounding does not mean zero charge on the sphere!)



PROBLEM 6 (Points 25)

A resistor is made from two concentric spheres with radii a and b between which there is a material with resistivity that varies with radius as $\rho(r) = \rho_0(r/a)^s$ where ρ_0 is a constant.

A battery with voltage V is hooked up to the resistor such that a current is injected into the resistor from the inner sphere, and removed from the outer sphere.

(a) (10pts) What must the exponent s be if the electric field between the spheres is constant?

(b) (6pts) What is the current, I in the circuit?

(c) (9pts) If we immerse the system in a vat with a mass M of water, specific heat c_W and initial temperature T_0, what will be the temperature of water as a function of time? You may assume that m is sufficiently large and T_0 is sufficiently far from the boiling point or freezing point of water such that there is no phase change that occurs.

PROBLEM 7 (Tot 25pts)

A real heat engine working between heat reservoirs at T_L and $T_H = 2T_L$ produces W of work per cycle for a heat input of Q_{in}. The heat output is labeled Q_{out}.

a) (8pts) What is the efficiency of this real engine? If the engine were a Carnot engine operating between the same two heat baths, what would the efficiency be? You can use the efficiency formula for a Carnot engine.

b) (8pts) Calculate the total entropy change of the universe for each cycle of the real engine.

c) (9pts) Calculate the total entropy change of the universe for a Carnot engine operating between the same two temperatures. Do not simply assert the answer, but provide a proof.

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$E = \frac{D}{2}nRT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$dE = -PdV + dQ$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For an adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2}R$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$dQ = TdS$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \text{ (For infinite wire)}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\vec{\nabla}V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$C = \frac{\epsilon A}{d} \text{ (parallel plate)}$$

$$C = \frac{2\pi\epsilon l}{\ln(r_a/r_b)} \text{ (cylindrical)}$$

$$C = 4\pi\epsilon\frac{r_a r_b}{r_a - r_b} \text{ (spherical)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junc.}} I = 0 \text{ (junction rule)}$$

$$\sum_{\text{loop}} V = 0 \text{ (loop rule)}$$

$$d\vec{F}_m = Id\vec{l} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{\mu} = NI\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r} \text{ (For infinite wire)}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$M = N_1 \frac{\Phi_1}{I_2} = N_2 \frac{\Phi_2}{I_1}$$

$$L = N\frac{\Phi_B}{I}$$

$$U = \frac{1}{2}LI^2$$

$$U = \int \frac{1}{2\mu_0} |\vec{B}|^2 dV$$

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$$

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}$$

(Cylindrical Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi}$$

(Spherical Coordinates)

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$

solves $\frac{dy}{dt} = -Ay + B$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$

solves $\frac{d^2y}{dt^2} = -Ay$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1 + x^2)^{-1/2} dx = \ln(x + \sqrt{1 + x^2})$$

$$\int (1 + x^2)^{-1} dx = \arctan(x)$$

$$\int (1 + x^2)^{-3/2} dx = \frac{x}{\sqrt{1 + x^2}}$$

$$\int \frac{x}{1 + x^2} dx = \frac{1}{2}\ln(1 + x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \frac{dx}{x} = \ln(x)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1 + x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1 + x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$