First Midterm Examination
Closed Books and Closed Notes
Answer all Three Questions for Maximum Credit

Question 1
A Particle in Motion on an Incline (20 POINTS)

As shown in Figure 1, a particle of mass m is free to move on a rough inclined plane. The contact between the particle and the plane has a coefficient of static friction μ_s and a coefficient of dynamic friction μ_k. The angle of inclination of the plane with the horizontal is denoted by β.

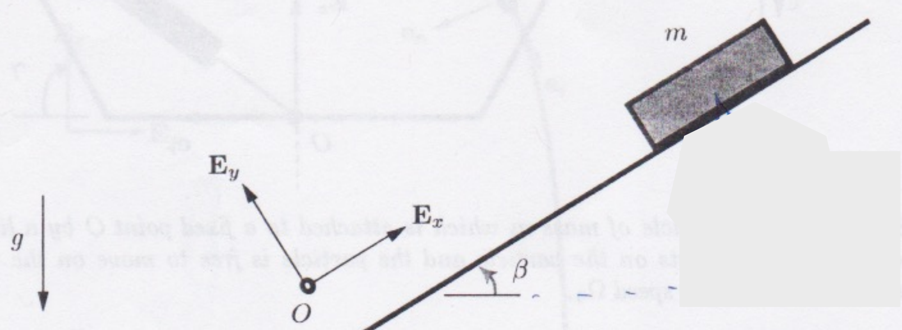

Figure 1: *Schematic of a particle of mass m which is free to move on a rough inclined plane.*

(a) Assume that the particle is moving on the inclined plane. Starting from the standard representation for the position vector,

$$\mathbf{r} = x\mathbf{E}_x + y_0\mathbf{E}_y + z_0\mathbf{E}_z, \tag{1}$$

where y_0 and z_0 are constants, establish expressions for the velocity vector $\mathbf{v}$ and acceleration vector $\mathbf{a}$ of the particle.

(b) Draw a freebody diagram of the particle in motion. Your freebody diagram should include a clear expression for the dynamic friction force.

(c) Suppose that the particle is in motion on the plane. Show that the differential equation governing the motion of the particle is

$$m\ddot{x} = -\mu_k ?? - mg??? \tag{2}$$

For full credit supply the missing ?? and ??? terms.

(d) Suppose that the particle is instantaneously at rest. Show how a criterion featuring μ_s and β can be established which, if satisfied, indicates that the particle will remain at rest.

Question 2

A Particle Moving on a Spinning Cone (20 POINTS)

As shown in Figure 2, a particle of mass m is attached to a fixed point O by a linearly elastic spring. The spring has a stiffness K and an unstretched length ℓ_0. The particle is also subject to a vertical gravitational force and is free to move on the inner surface of a rough truncated cone. The rotational speed Ω_0 of the cone is constant.

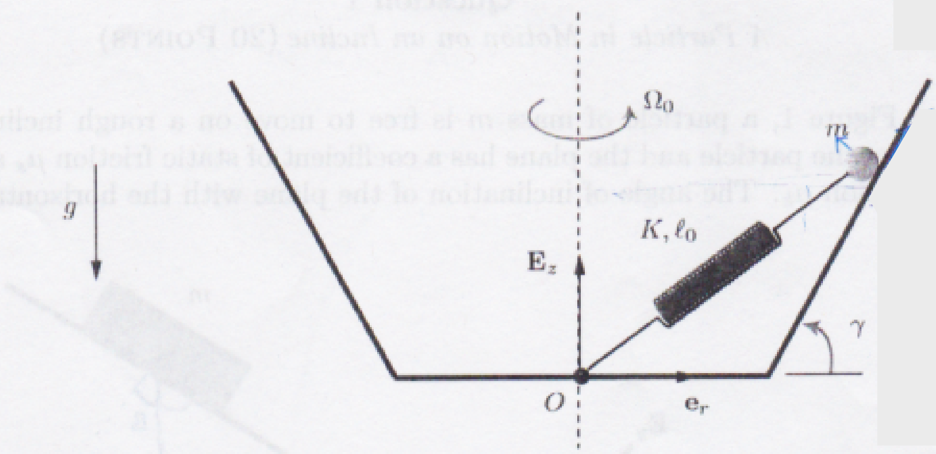

Figure 2: *Schematic of a particle of mass m which is attached to a fixed point O by a linearly elastic spring. A gravitational force $-mg\mathbf{E}_z$ acts on the particle and the particle is free to move on the rough surface of a cone that is spinning at a constant speed Ω_0.*

(a) Suppose the particle is in motion on the spinning cone. Starting from the standard representation for the position vector

$$\mathbf{r} = r\mathbf{e}_r + (r - r_0)\tan(\gamma)\,\mathbf{E}_z, \tag{3}$$

where r_0 is constant, establish expressions for the velocity vector $\mathbf{v}$ and acceleration vector $\mathbf{a}$ of the particle.

(b) Draw a freebody diagram of the particle. In addition to expressions for the normal vector $\mathbf{n}$ and unit tangent vectors $\mathbf{t}_1$ and $\mathbf{t}_2$ to the surface of the cone, your freebody diagram should include clear expressions for the normal force, spring force, and the dynamic friction force.

(c) Suppose that the particle is stationary on the spinning cone. Show that the normal force $\mathbf{N}$ and friction force $\mathbf{F}_f$ acting on the particle have the representations

$$\mathbf{N} = (??)\,\mathbf{n}, \qquad \mathbf{F}_f = (???)\,(\cos(\gamma)\mathbf{e}_r + \sin(\gamma)\mathbf{E}_z)\,. \tag{4}$$

For full credit supply the missing ?? and ??? terms.

Question 3

Aerosol Dynamics (20 Points)

An aerosol is a suspension of particles in a gas. A drag force, know as Stokes' force features prominently in the dynamics of the suspended particles (particulates). This force has the representation

$$\mathbf{F}_{\text{Stokes}} = -mc\mathbf{v}, \tag{5}$$

where c is a positive constant. In this problem, the dynamics of a single suspended particle subject to a vertical gravitational force $-mg\mathbf{E}_z$ and Stokes' force is considered.

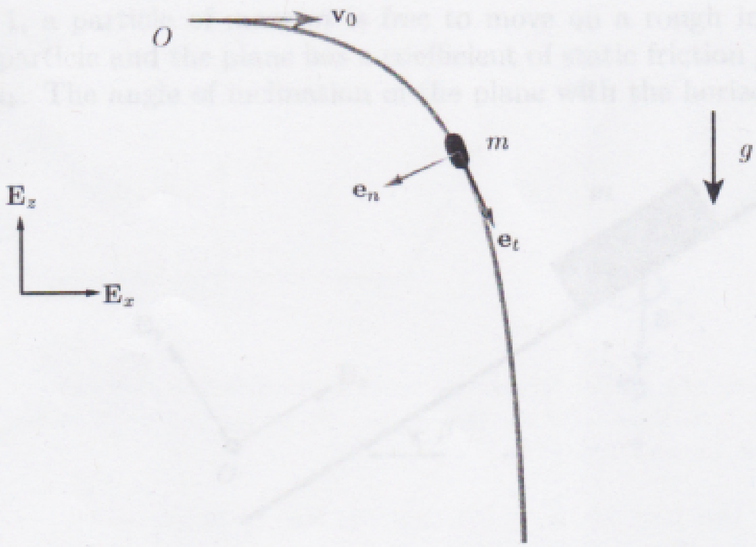

Figure 3: *Path of a particulate ejected with an initial velocity* $\mathbf{v}_0$ *from the nozzle of an aerosol can.*

(a) Starting from the representation $\mathbf{r} = \mathbf{r}(s(t))$ for the position vector of a particle, establish the representation

$$\mathbf{a} = \dot{v}\mathbf{e}_t + \kappa v^2 \mathbf{e}_n. \tag{6}$$

(b) Referring to Figure 3, consider a particle of mass m which is ejected from the nozzle of an aerosol can with the following initial position vector and initial velocity vector:

$$\mathbf{r}(t=0) = \mathbf{0}, \qquad \mathbf{v}(t=0) = \mathbf{v}_0 = v_{0_x}\mathbf{E}_x + v_{0_y}\mathbf{E}_y + v_{0_z}\mathbf{E}_z. \tag{7}$$

The equation of motion for the particle is

$$\dot{\mathbf{v}} = -c\mathbf{v} - g\mathbf{E}_z. \tag{8}$$

(i) Verify that the path of the particle during the subsequent motion is given by

$$\mathbf{r}(t) = \frac{1}{c}\left[1 - e^{-ct}\right]\mathbf{v}_0 + \frac{g}{c}\left[\frac{1}{c}\left(1 - e^{-ct}\right) - t\right]\mathbf{E}_z. \tag{9}$$

(ii) What is the reach of the spray? [I.e., the maximum distance from O that the particle travels in the horizontal plane].

(iii) What is the terminal velocity vector of the particle?

(iv) Show that the terminal curvature of the path of the particle is zero.