Luk/Smoot Midterm 2: Problem 1 Grading Scheme

Part a: 5 pts total

Point breakdown

Neutral matter made up of charged constituents (1 pt)

Neutral matter attracted to charged matter because of: (3 pts)

separation of charge (polarization) on neutral matter (1 pt)

induced by proximity to charged matter (1 pt)

with opposite charge on face nearest to the charged matter (1 pt).

There are attractive and repulsive forces, but since the oppositely charged face is closer, the net force is attractive (1 pt).

Note: Talking about charge transfers could be worth up to 2 points depending on how cogent the discussion, but in most cases this sort of argument would imply that neutral matter would be repulsed by any charged matter.

Part b: 5 pts total

Point breakdown

Power tools run poorly because: (3 pts)

Long cords have non-negligible resistance ($R = \rho L/A$) (2 pt)

which raises effective resistance and thus lowers current (1 pt)

OR

lowers effective voltage of outlet (1 pt).

To fix this problem, we can: (up to 2 pts)

use thicker cords (larger A makes R smaller) (2 pt)

splice two cords together in parallel (same as above) (2 pt)

use a variac or some sort of transformer system (2 pt)

use a metal with higher conductivity (1 pt)

(somehow) use a shorter cord (1 pt).

Note:

Of all the solutions, using a thicker cord is obviously the most practical.

The current being AC is essentially a red herring.

The power loss in the wire is irrelevant. A power outlet is an infinite source of power; it can provide (in the ideal world of physicists) infinite amounts of current. There *is* less power available to the power tools, but this is because the current they draw is less or the effective voltage they have available is less (these two things are equivalent).

Part c. 5 pts total

Point breakdown

Charge lies on the surface of the conductor. (1 pt)

Conductor (and its surface) is equipotential (1 pt)

$V = kQ/R$ for perfect spheres, so we expect Q to be proportional to the radius of curvature. (1 pt)

Surface charge σ is $Q/(4\pi R^2)$. Using the fact that Q is proportional to R, σ is inversely proportional to R so the charge density will be higher on the smaller spherical surface. (2 pts).

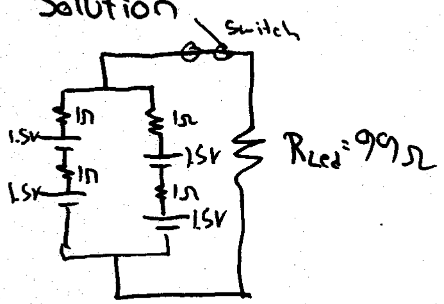
Note: There is indeed more charge on the left side (approximately twice as much since the radius is twice as large) but the density is still larger on the right. If no progress was made on the above track, extra points still earned by the following: Arguing that there was more charge on the left on physical grounds was worth 1 pt, OR realizing that the surface charge had to be nonuniform was worth 1 pt. If you could convincingly argue by virtue of vanishing E field within the conductor that the charge density had to be larger on the right, you got up to 2 pts out of the last 4.

Remark: It is **NOT** a property of conductors that the surface charge is uniform. Rather, it is a property that charges are free to move and therefore E=0 inside and V is constant. The surface charge will be whatever it must be for this to be achieved. Only in the cases of highly symmetric surfaces (spherical, cylindrical, or planar) will the surface charge be uniform.

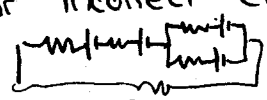
2. [15 points] My remote control contains four AAA batteries. Two batteries are in series (called them a pair), and the two pairs are connected in parallel. Each AAA battery provides 1.5 V EMF and has an internal resistance of 1 Ω. When a button is pressed the battery combination is connected across an LED (light emitting diode) with an equivalent resistance $R_{\text{LED}} = 99\ \Omega$.

(a) [3 points] Draw the equivalent circuit diagram of the full system, label the EMFs, internal resistances, and the LED resistance.

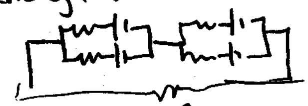
Solution



−2 for incorrect circuit diagram



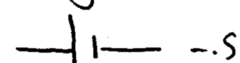
−2



−1.5

−.5 for not labeling R internal

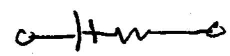
or doing



$V = \frac{3}{2}$, $R_{int} = 1\Omega$

−.5 for not drawing circuit elements right

or for not making a circuit

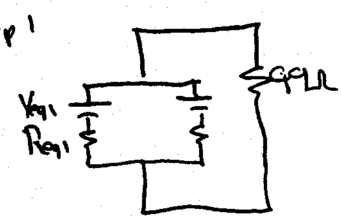


(b) [3 points] Draw the reduced equivalent circuits for a single battery with internal resistance, and the LED resistance. Label the EMF and resistances with their values.

Solution

Scheme (based on circuit from part a.)

step 1



$V_{eq1} = 1.5V + 1.5V = 3V$

$R_{eq1} = 1\Omega + 1\Omega = 2\Omega$

+1.5 for finding Rint

+.5 for work if wrong answer & good wo

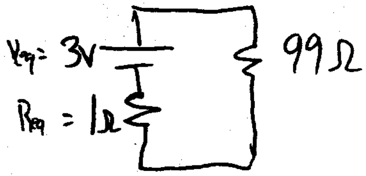
+1.5 for finding Veq

0 if wrong and no work

.5 if wrong and good work

−1 for circuit diagram incorrect or other error

step 2

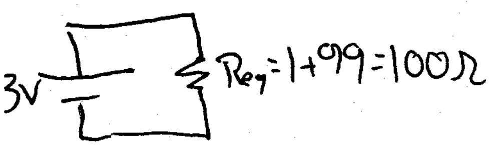


$V_{eq} = 3V$

$R_{eq} = \frac{1}{\frac{1}{\ }+\frac{1}{\ }} = 1\Omega$

(c) [3 points] Draw the reduced equivalent circuits with a single EMF and one resistance. Again, label the EMF and resistance with their values.

Solution



Scheme (based on soln. to part b.)

+1.5 for correct voltage (same as part b)

+1.5 for correct R_{eq} (+.5 if incorrect answer & good work)

−1 for misc. errors (drawing inc

(d) [3 points] Calculate the current through the LED and power dissipated at the LED.

Solution

$$I = \frac{V}{R} = \frac{3V}{100\Omega} = .03\,A$$

$$P = I^2 R = (.03)^2 \cdot 99\Omega = .088\,W$$

+.5 calculate current with form of $V = IR$

+1 use $R_{eq} = 100$ (or answer from c)

+.5 calculate Power with $P = \frac{V^2}{R}$ or $P = IV$ or $P = I^2R$

+1 use proper values for power calc. (either find voltage drop across 99Ω or use $i^2 \cdot 99$

(e) [3 points] What is the current through each battery? How much power does the EMF in each battery provide?

Solution

$I \rightarrow I/2$

$\quad \rightarrow I/2$

$$I_{bat} = \frac{I}{2} = .015\,A$$

$$P_{bat} = IV = .015 \cdot 1.5 = .0225$$

or

$$P_{bat} = \frac{P_{tot\,dis}}{4} = \frac{3^2/100}{4} = \frac{9}{400}$$

Scheme

+.5 for current calc or realizing $I/2$

+1 for plugging in proper values for current calc.
ie not $I = \frac{3V}{1\Omega}$

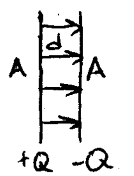
+.5 for Power equation and attempt to use it

+1 for plugging in proper values.

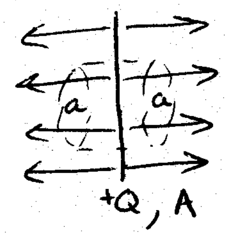
3. [20 points] A parallel-plate capacitor has plates of area A and spacing d between the plates. It is charged with charge Q at the beginning, and is then disconnected from the battery.

throughout, (−1) for math errors

(a) [5 points] Apply Gauss's law to determine the electric field **E** between the plates. Fringe field is ignored.



Apply Gauss's law to one plate:



$$\oint\!\!\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\cancel{A}) = \frac{Q}{A\varepsilon_0}\cancel{A}$$

$$E_+ = \frac{Q}{2A\varepsilon_0}$$

(likewise for E_-)

fields of positive & negative plates add in the middle, so

$$E = E_+ + E_- = \frac{Q}{A\varepsilon_0} \text{ from } +Q \text{ to } -Q$$

** Important Note **
(−0) points were given if no understanding was shown.
example: $\oint\!\!\oint E\cdot da = \frac{q_{enc}}{\varepsilon_0}$
$EA = \frac{Q}{\varepsilon_0}$, $E = \frac{Q}{A\varepsilon_0}$. (+0)

(+5) for perfect answer, with appropriate Gaussian surface. Find E as above, with superposition, or use Gauss's law on the 2-plate configuration & explain clearly why the flux is only through one face of surface. (−1) without explanation.

(+3) for finding E of single plate, explicitly. (+1) for drawing an appropriate Gaussian surface

(b) [5 points] Starting from the electric field found in part (a), determine the potential difference between the plates.

$$V = -\int \vec{E}\cdot d\ell = -\int_0^d \frac{Q}{\varepsilon_0 A}\,d\ell = -\frac{Q}{\varepsilon_0 A}\,\ell\Big|_0^d = \frac{-Qd}{\varepsilon_0 A}$$

(+5) for perfect answer. No points lost because of the overall sign.

(+3) for skipping integration and using formula $V = Ed$.

(−1) for wrong/missing limits of integration

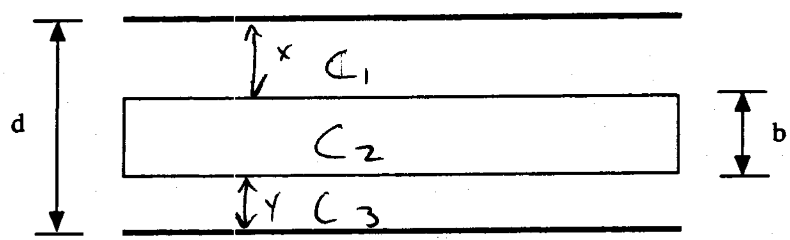
(c) [1 points] Calculate the capacitance of this parallel-plate capacitor.

(+1) for $Q = CV$, $C = \dfrac{Q}{V} = \dfrac{\cancel{Q}}{\left(\dfrac{\cancel{Q}d}{A\varepsilon_0}\right)} = \dfrac{A\varepsilon_0}{d}$

points were not deducted if student arrived at the wrong answer because answer to (b) was wrong.

no points were given for a negative value of C

(d) [4 points] A dielectric slab of thickness b ($b < d$) is then positioned between the plates.If the dielectric constant of the slab is κ, find the capacitance of the system after the slab is in place as shown in Figure 2



$x + y = d - b$

Figure 2: Figure for problem 3

First method

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{x}{A\varepsilon_0} + \frac{b}{KA\varepsilon_0} + \frac{y}{A\varepsilon_0}} = \frac{A\varepsilon_0}{d - b + \frac{b}{K}}$$

Second method

$$C = \frac{Q}{V} = \frac{Q}{-\int \vec{E}\cdot d\vec{l}} = \frac{Q}{\frac{\cancel{Q}(d-b)}{A\varepsilon_0} + \frac{\cancel{Q}b}{KA\varepsilon_0}} = \frac{A\varepsilon_0}{d - b + \frac{b}{K}}$$

(+4) perfect answer

(+1) for making a stron attempt on the right track but messing it up / not finishing

(+0) for writing down a random answer with no / wrong explanation

(e) [5 points] How much work was done by the person who moved the slab from very far away to its final position between the plates?

$$W_{\text{by person}} = U_f - U_i$$

$$U_{cap} = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

$$W = \frac{Q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{Q^2}{2}\left(\frac{\cancel{d} - b + \frac{b}{K}}{A\varepsilon_0} - \frac{\cancel{d}}{A\varepsilon_0}\right)$$

$$= \frac{Q^2}{2A\varepsilon_0}\left(\frac{b}{K} - b\right)$$ this is a negative #

(+4) for perfect answer

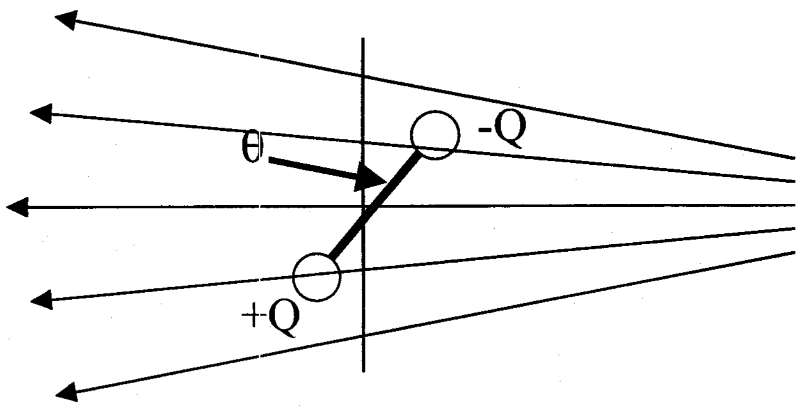
(+1) for $W = U_f - U_i$, but messing up (ie. V constant or $U = qV$)

(-2) for $W = U_i - U_f$

(-2) for messing up the equation for U_{cap}. This does not include $U = qV$ because this is

Problem 4 [20 points]

Given: a dipole in a *nonuniform* electric field.



(a) [5 points] Is there a torque on the dipole? Explain your answer. If yes, find the direction and magnitude.

First, the explanation: Yes, there is a torque because the positive charge feels a force to the left, and the negative charge feels a force to the right. These forces will spin the charge on the axis through the center. In other words, the dipole "wants" to align itself with the electric field, at which point it no longer feels a torque. This torque exists even when the field is *uniform*.

The torque on a dipole in an external field is given by the formula on the formula sheet: $\vec{\tau} = \vec{p} \times \vec{E}$. We can use this formula to get both the magnitude and direction. The magnitude of the cross product is $|\vec{\tau}| = |p| \cdot |E| \sin\phi$, where ϕ is the angle between the dipole vector and the electric field vector. From the diagram, we can see that this angle is 90°-_. As we announced during the exam, the magnitude of the electric field at the center of the dipole is E_0. Since the dipole is small, we can just assume that this is the field that creates the torque. So,

$$|\vec{\tau}| = |\vec{p}| \cdot |\vec{E}| \sin(90^o - \theta) = Qd \cdot E_0 \cos\theta .$$

The direction, given by the right hand rule, is *into the page*. While the rotation that results is clockwise, the direction of the torque vector is into the page. An alternate method is to calculate the torque directly by summing the torques on each charge about the center of the dipole (you'll need to multiply the force on each by cos_ to pick out the perpendicular component). Both methods give the same answer.

Grading:
2 points for explanation. 1 point is given if the justification involves only the equation and ignores the physics behind it.
2 points for the correct magnitude, 1 point if there is one mistake.
1 point for correct direction.

(b) [5 points] Find the potential at the center of the dipole due to the dipole. Assume V = 0 at infinity.

For this part, we can ignore the background field. Since the electric potential is set to zero at infinity, we can use the formula for a point charge, $V = q/(4\pi\varepsilon_0\rho)$, to figure the electric potential due to each charge. We then use superposition and add these contributions. The distance to each charge from the center is simply $r = d/2$. Thus,

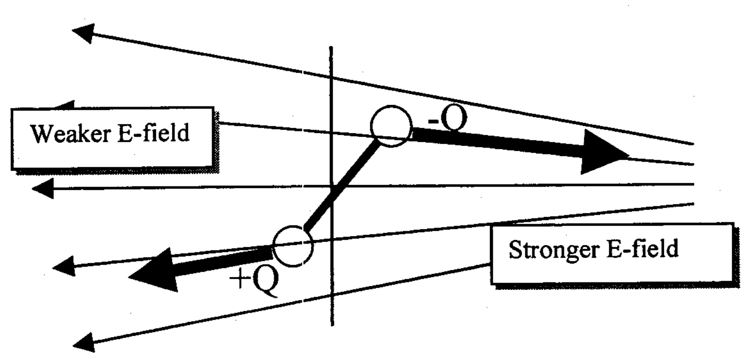
$$V_{center} = \frac{+Q}{4\pi\varepsilon_0\left(\frac{d}{2}\right)} + \frac{-Q}{4\pi\varepsilon_0\left(\frac{d}{2}\right)} = 0.$$

The most common mistake was using the formula for the potential *energy* of a dipole in an external electric field. For one, that gives potential energy, not electric potential. Also, it depends on the external field which we need to ignore.

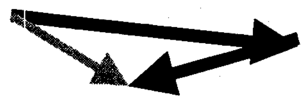
Grading:
Most people either got it or didn't. Finding the potential energy in the external field was worth 0 points. It was worth 1 point if you divided by q to get something with units of electric potential, even though that expression doesn't really make sense.
Integrating the wrong electric field or trying to use Gauss' Law received 0 points.
Having nothing but the correct answer written down was worth 2 points. 2 points were also given if one made a math error in the calculation given above.

(c) [10 points] Is there a net force on the dipole? Explain your reasoning. If there is, give the magnitude and direction.

Yes, there is a net force. Sure, dipoles feel no net force if the field is uniform. In that case, the force vectors on each charge will cancel exactly. However, this is a *nonuniform* field. So, one can imagine that those two force vectors will not cancel each other out. Also, the field strength is different on the two charges. The negative charge is in a region with denser field lines. Thus, the electric field is higher there than at the positive charge. I've drawn force vectors on the diagram below.



Note that the vectors are parallel to the E-field (they should be, at least), and that the force vector on the negative charge is longer because the E-field is greater there. If we add the vectors up, we get a resultant that points to the *right* and *down.*



The dominant force is the one pointing to the right. If we imagine that the electric field is that of a positive point charge located to the right (off the page), the negative charge is attracted and the positive charge is repelled. However, the negative charge is closer and is attracted more than the positive charge is repelled.

Finding the magnitude of the force is difficult because very little information is provided. To get an answer, you would have to introduce quantities of your own and then calculate. I accepted many different answers as long as they were consistent and physically correct. Below, I give my solution.

I assume that the field can be expanded as a uniform background with a linear correction to it. If you're familiar with Taylor series from calculus, I'm basically taking the first two terms of the Taylor series and throwing away the rest because they're very small. This is valid because the field doesn't change much over the length of the dipole. So, the electric field becomes (in the vicinity of the dipole)

$$\vec{E} = -\varepsilon_x \Delta x \hat{i} + \varepsilon_y \Delta y \hat{j} - E_0 \hat{i} \ ,$$

where $_x$ and $_y$ are the linear corrections, E_0 is the background field pointing to the left, and the origin of my coordinates is at the center of the dipole. I assume the linear coefficients are positive; therefore, there has to be a minus sign on the $_x$ (so we have a stronger, left-pointing field to the right of the vertical line). We can throw out the uniform field because we know that there will be no net force because of it. Now, let's consider the force on each charge:

$$\vec{F}_{+Q} = -\varepsilon_x \frac{Q(-d)}{2} \sin\theta \cdot \hat{i} + \varepsilon_y \frac{Q(-d)}{2} \cos\theta \cdot \hat{j}$$

$$\vec{F}_{-Q} = -\varepsilon_x \frac{(-Q)d}{2} \sin\theta \cdot \hat{i} + \varepsilon_y \frac{(-Q)d}{2} \cos\theta \cdot \hat{j} \ .$$

Be very careful with signs. I then add up these terms to get the net force:

$$\vec{F}_{TOTAL} = \varepsilon_x Qd \sin\theta \cdot \hat{i} - \varepsilon_y Qd \cos\theta \cdot \hat{j} \ .$$

This result gives us the behavior that we deduced before. The net force points in the positive x-direction and in the negative y-direction (to the right and down). To get the magnitude, we simply take the magnitude of this vector.

$$\left|\vec{F}_{TOTAL}\right| = \sqrt{(\varepsilon_x Qd \sin\theta)^2 + (\varepsilon_y Qd \cos\theta)^2}$$

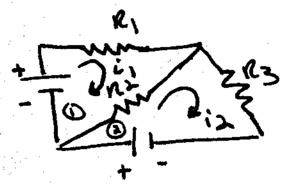
Grading:
Explanation: 2 points. I wanted to see something about the nonuniform field or that the negative charge was in a region with greater electric field. If you forgot that the field is not uniform, and said there was no net force, you got 1 point. Saying something like "having a nonzero torque means that there is a force because $\tau = r \times F$" is worth 0 points. The question asks for *net* force.
Direction: 3 points. Saying the dipole feels the net force to the right is worth 2 points and saying that it also moves down is worth 1 point. The force directed to the right, as I said, is the stronger effect.
Magnitude: 5 points. It's hard to give an objective scheme. If you gave an attempt that was wrong, I usually gave it 1 point. Most of the answers that got 1 point here forgot to separate out each component and then figure out the magnitude. I awarded the rest of the 5 points on a holistic scale.

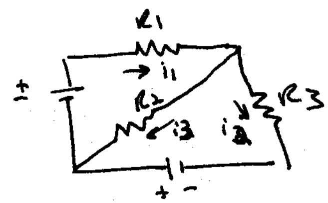
General Comments about the problem:

The mean score on this problem was about 5.2 out of 20. So, don't get too upset if your score is in the single digits. I did not give any credit for simply writing an equation down. The equations needed were on the formula sheet with explanations of what they were, so it didn't necessarily show lots of understanding to just copy them down.

Grading Scheme for Problem 5, Midterm 2.

5a. Goal: To apply Kirchoff's loop/node laws to the circuit in order to obtain a set of independent equations for the current.





Loop Laws:

① $\mathcal{E}_1 - R_1 i_1 - R_2(i_1 - i_2) = 0$

② $\mathcal{E}_2 - R_2(i_2 - i_1) - R_3 i_2 = 0$

Loop Law

$\mathcal{E}_1 - R_1 i_1 - R_2 i_3 = 0$

$\mathcal{E}_2 + R_2 i_3 - R_3 i_2 = 0$

Node Law

$i_1 = i_2 + i_3$

$\Downarrow$

$\mathcal{E}_1 - R_1 i_1 - R_2(i_1 - i_2) = 0$

$\mathcal{E}_2 - R_2(i_2 - i_1) - R_3 i_2 = 0$

Points:

6: A set of currents are labeled (with directions) on the circuit. Kirchoff's loop/node laws are applied, and these are consistent with the current labels given. Either 2 or 3 equations should be written depending on whether each R is given its own current (I1, I2, I3) or whether superposition of currents is used (I1, I2).
5: A set of equations is written but a sign error (current direction) occurs.
4: Two independent equations involving current, but a 3rd is absent or incorrect.
3: One correct equation appears, and the student tries unsuccessfully to set up remaining equations.
2: One correct equation.
1: Massive confusion.
0: Blank.

5b. Goal: To solve the written independent equations for current through R1 and find V1 across RI.

$$\mathcal{E}_1 - R_1 i_1 - R_2 i_1 + R_2 i_2 = 0 \quad \Rightarrow \quad i_2 = \frac{-\mathcal{E}_1 + R_1 i_1 + R_2 i_1}{R_2}$$

$$\mathcal{E}_2 - R_2 i_2 + R_2 i_1 - R_3 i_2 = 0$$

$$\mathcal{E}_2 - \cancel{R_2}\left(\frac{-\mathcal{E}_1 + R_1 i_1 + R_2 i_1}{\cancel{R_2}}\right) + R_2 i_1 - R_3\left(\frac{-\mathcal{E}_1 + R_1 i_1 + R_2 i_1}{R_2}\right) = 0$$

$$\mathcal{E}_2 + \mathcal{E}_1 + \frac{\mathcal{E}_1 R_3}{R_2} - i_1\left(R_1 + \frac{R_3 R_1}{R_2} + R_3\right) = 0$$

$$i_1 = \frac{\mathcal{E}_2 + \mathcal{E}_1 + \frac{\mathcal{E}_1 R_3}{R_2}}{R_1 + \frac{R_3 R_1}{R_2} + R_3} = \boxed{\frac{\mathcal{E}_2 R_2 + \mathcal{E}_1 R_2 + \mathcal{E}_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

$$\boxed{V_1 = i_1 R_1}$$

Points:

7: I1 is found correctly by whatever algebra is necessary.
6: The solution strongly resembles I1 but is wrong by 1-2 sign errors.
5: Solution does not strongly resemble correct solution but the units are correct (Much algebraic struggling has occurred).
4: Solution does not strongly resemble correct solution and units are wrong (not of the form I=V/R)
3: A lot of math occurs but I2 or I3 remain in the solution (student runs out of time in the calculation).
2: I3 or I2 is in the answer and student has not tried very hard to solve the equations.
1: Massive confusion. (e.g. Student tries to solve for R1).
0: Blank or no indication of a plan of solution.

2: The voltage across resistor 1 is correctly identified as V1=R1*I1. I1 does not need to be correct.
0: Voltage not found or incorrect expression given.

Yuki

6. [15 points] The flat circular disk of charge in Figure 5 has a radius of R and a uniform surface charge density σ.

+1 for recognizing that $V(y)$ has to be with respect to $V(0)=0$.

+1 for correctly taking this into account.

+1 for $dq = \sigma 2\pi \rho\, d\rho$

+1 for correctly setting up $V'(y)$ or ΔV

+1 for checking $V(0)=0$ for your answer.

+1 for recognizing $V(y)<0$.

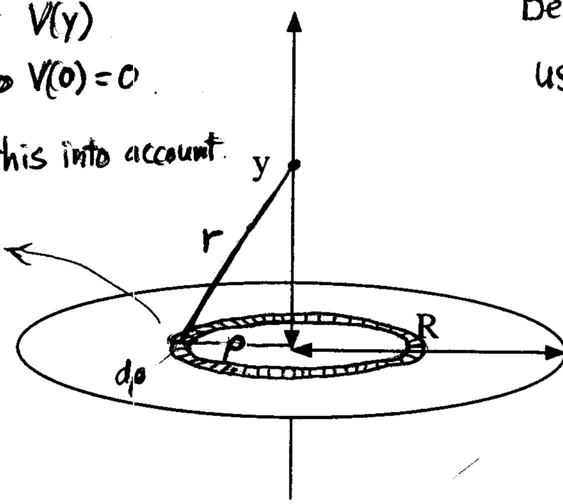
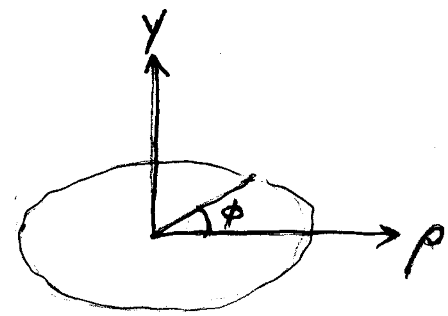


Figure 5: Figure for problem 6

Because of the cylindrical symmetry, use cylindrical coordinates:



$\vec{E}$ is not uniform, so Gauss's law only works for $y \ll R$.

-1 for answers w/ wrong dimensions/units ⟵ always check this!

(a) [6 points] Find the potential $V(y)$ at a point that is at a vertical distance y from the center of the disk if $V(0)=0$.

First, in a reference where $V'(\infty)=0$:

$$dV' = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \qquad \begin{array}{l} \longleftarrow dq = \sigma 2\pi\rho\, d\rho \quad \text{- charge in the thin ring of width } d\rho \\ \longleftarrow r = \sqrt{y^2+\rho^2} \quad \text{- distance to the charge element} \end{array}$$

$$= \frac{\sigma}{2\varepsilon_0}\frac{\rho}{\sqrt{y^2+\rho^2}}d\rho$$

$$V'(y) = \int_{disc} dV'$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{\rho}{\sqrt{y^2+\rho^2}}d\rho \qquad \text{Let } u \equiv y^2+\rho^2 \Rightarrow du = 2\rho\, d\rho$$

$$= \frac{\sigma}{2\varepsilon_0}\int_{y^2}^{y^2+R^2}\frac{1}{2\sqrt{u}}du$$

$$= \frac{\sigma}{2\varepsilon_0}\sqrt{u}\Big|_{y^2}^{y^2+R^2}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{y^2+R^2}-|y|\right)$$

So, $V'(0) = \frac{\sigma}{2\varepsilon_0}R$

If $V(0)=0$,

$$V(y) = V'(y) - V'(0)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{y^2+R^2}-|y|-R\right) < 0$$

Check $V(0)=0$ ✓

+1 for recognizing that it's <u>lower</u> than $V(0)$.

+1 for understanding that it's lower by a <u>finite</u> amount.

+1 for correctly setting up the difference in potential.

+1 for correct value.

Always check!! (It's physics - NOT Math!!)

-1 for not checking if your answer makes sense (for example, 0 or ∞)

-1 for answers with wrong dimensions/units.

(b) [4 points] What is the potential at y equal to infinity?

If $V(0) = 0$, then $V(\infty)$ is lower by $V'(0)$

So, $V(\infty) = -V'(0)$

+1 for symmetry argument $= -\frac{\sigma}{2\varepsilon_0} R$

+1 for correctly taking $\vec{\nabla} V$

+1 for understanding and noting the direction ($\hat{y}$)

+1 for making sure $\vec{E}(\infty) = 0$.

+1 for making sure $\vec{E}(0_+) = \frac{\sigma}{2\varepsilon_0}\hat{y}$

right above the surface

(b) [5 points] Determine the associated electric field $\mathbf{E}$ using the relation $\mathbf{E} = -\nabla V$.

Along the axis:

Because of the cylindrical symmetry,

the electric field will only have a vertical component:

$$\vec{E} = -\vec{\nabla} V = -\left(\cancel{\frac{\partial V}{\partial \rho}\hat{\rho}} + \cancel{\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\phi}} + \frac{\partial V}{\partial y}\hat{y}\right)$$

-1 for answers with wrong dimensions/units.

$$= -\frac{\partial V}{\partial y}\hat{y} = -\frac{\partial}{\partial y}\left[\frac{\sigma}{2\varepsilon_0}\left(\sqrt{y^2+R^2} - |y| - R\right)\right]\hat{y} \overset{\text{for } y>0}{=} -\frac{\sigma}{2\varepsilon_0}\left(\frac{y}{\sqrt{y^2+R^2}} - 1\right)\hat{y}$$

[$E(y<0) = -\vec{E}(|y|)$ opposite direction]

Check $E(\infty) = 0$, $\vec{E}(0) = \frac{\sigma}{2\varepsilon_0}\hat{y}$ $\longleftarrow$ $\boxed{\vec{E}(y_{>0}) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{y}{\sqrt{y^2+R^2}}\right)\hat{y}}$