PHYSICS 7B, Lectures 1 & 3 – Spring 2015
Midterm 1, C. Bordel
Monday, February 23, 2015
7pm-9pm

Make sure you show all your work and justify your answers in order to get full credit.

<u>Problem 1</u> – Thermal expansion (20 pts)

An alcohol thermometer is made of a cylindrical tube of inner diameter d_0 and a bulb of volume V_0 at room temperature (T_0). The volumetric coefficients of thermal expansion are β_{al} and β_g for the alcohol and glass respectively, with $\beta_{al} >> \beta_g$.
Assume that the thickness of the Pyrex glass is negligible.

a- If the volume of the bulb is much bigger than that of the tube, determine the change in volume of the inside of the thermometer when the temperature is increased from T_0 to T.
b- Determine the change in volume of the alcohol between the same temperatures.
c- What is the change in height of the column of alcohol between T_0 to T?
d- What is the change in height of the column of alcohol between the same temperatures if the change in volume of the tube cannot be neglected?

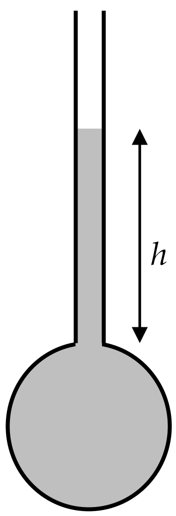

Figure 1

<u>Problem 2</u> – Equipartition of energy & Conductive heat transfer (20 pts)

First part: Equipartition of energy

Two separate ideal gases of same molecular mass m and maintained at the same temperature T are compared. Assume that $T \in [100\text{ K}; 1000\text{ K}]$. There is 1 mole of gas in each container of same volume V, one is monatomic and the other is diatomic.

a- How many degrees of freedom does each gas molecule have? How does that affect the average translational kinetic energy of each gas molecule?

b- Calculate the average rotational kinetic energy and average internal energy per molecule of each type of gas.

Second part: Conductive heat transfer

In a very schematic view, the Earth can be modeled by a hot inner core, in the form of a sphere of radius R_c and constant temperature T_c, surrounded by a thick layer (spherical shell) extending from R_c to R_s, that has thermal conductivity k. The temperature at the Earth's surface, T_s, is assumed to be constant, and the outside layer is supposed to be homogeneous. Let r be the radial distance from the center of the Earth.

c- What are the assumptions you can make regarding the rate of conductive heat flow $P(r,t)$? Explain your reasoning.

d- Under the previous assumptions, calculate the temperature profile $T(r)$ throughout the outside layer of the Earth.

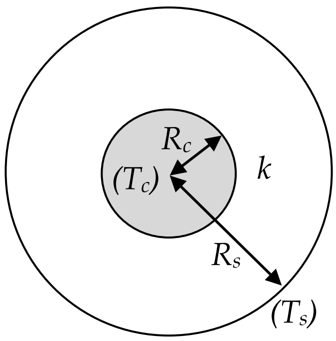

Figure 2

<u>Problem 3</u>– Heat engine (20 pts)

An ideal diatomic gas containing an unknown number of moles is used as the working substance of a heat engine operating according to the following cycle.
All answers should be in terms of V_a, V_b, P_a, T_a.

a- Explain why the cycle represented on the PV diagram is that of a heat engine. Calculate the pressure at points *b* and *c*, as well as the temperature at point *c*.

b- Calculate the net work done by the gas over one full cycle.

c- Identify the process(es) resulting in a heat input and calculate Q_{in}.

d- Determine the engine's efficiency and compare it with the efficiency of an ideal engine operating between T_a and T_c by calculating the ratio of the two efficiencies.

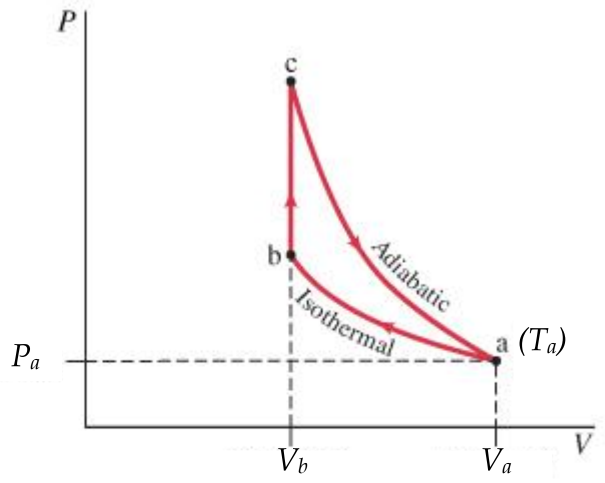

Figure 3

<u>Problem 4</u> – Calorimetry & Entropy (20 pts)

A block of silicon of mass m_{Si}, specific heat C_{Si} and initial temperature T_{Si} is immersed in liquid mercury (mass m_{Hg}, specific heat C_{Hg}) at initial temperature T_{Hg} in a Styrofoam container.

a- Explain, using the concept of thermodynamic equilibrium, why a thermodynamic process that involves 2 objects having different temperatures that are put in thermal contact with each other is not reversible.

b- Calculate the final temperature (T_f) of the system.

c- Calculate the total entropy change of the system {silicon + mercury} as it evolves from the initial situation to the thermodynamic equilibrium.

d- What sign do you expect for this entropy change? Explain.

<u>Problem 5</u> – Electrostatics (20 pts)

Two parallel circular rings of same radius R are separated by a distance 2ℓ along the x axis. The rings carry opposite uniform charge distributions, λ and $-\lambda$ ($\lambda > 0$).

a- Calculate the electric field $\vec{E}(O)$ at the center.

b- Calculate the electric field $\vec{E}(x)$ at any point on the symmetry axis.

c- Determine the asymptotic expansion of $E(x)$ for large x.

d- Graph the function $E(x)$ that was calculated in part (c).

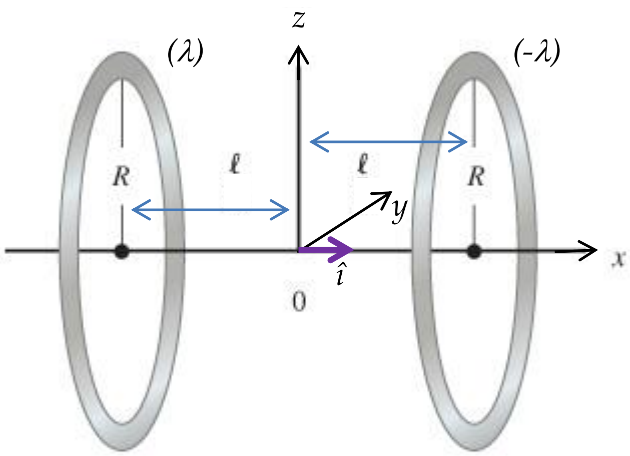

Figure 4

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$E_{int} = \frac{d}{2}NkT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$\Delta E_{int} = Q - W$$

$$dE_{int} = dQ - PdV$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For a reversible adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2}R$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W_{net}}{Q_{in}}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$\Delta S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0 \text{ (For irreversible processes)}$$

$$\oint dE = \oint dS = 0$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kdQ}{r^2}\hat{r}$$

$$\lambda = \frac{dQ}{dl} \qquad \sigma = \frac{dQ}{dA} \qquad \rho = \frac{dQ}{dV}$$

$$\overline{g(v)} = \int_0^\infty g(v)\frac{f(v)}{N}dv$$
$$\text{(} f(v) \text{ a speed distribution)}$$

$$\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n}e^{-ax^2}dx = \frac{(2n)!}{n!2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1}e^{-ax^2}dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2}dx = \ln(x+\sqrt{1+x^2})$$

$$\int (1+x^2)^{-1}dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2}dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2}dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)}dx = \ln\left(\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)}dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$