Problem 1

a) Since the volume of the tube is negligible, we only need to calculate the expansion of the bulb. Almost directly from the formula sheet,

$$\Delta V_g = \beta_g V_0 (T - T_0).$$

b) Again we neglect whatever volume of alcohol is in the tube, so

$$\Delta V_{al} = \beta_{al} V_0 (T - T_0).$$

c) Because $\beta_{al} \gg \beta_g$, the expansion of the thermometer can be ignored. Using the result from part (b) and simple geometry,

$$\Delta V_{al} = \frac{\pi d^2}{4} \Delta h = \beta_{al} V_0 (T - T_0).$$

The desired change in the height of the alcohol column is therefore given by,

$$\Delta h = \beta_{al} V_0 (T - T_0) \frac{4}{\pi d^2}.$$

d) When the volume of alcohol in the thermometer can't be ignored, this volume also expands by

$$\Delta V = \beta_{al} \frac{\pi d^2}{4} h_0 \Delta T$$

and the total height change becomes

$$\Delta h = \beta_{al} \left(\frac{4}{\pi d^2} V_0 + h_0 \right) (T - T_0).$$

Problem 2

a) The monatomic gas has $d = 3$, while the diatomic gases have $d = 5$. The average translational kinetic energy is given by $\frac{3}{2}k_B T$ for both gases.

b) The monatomic gas has no rotational kinetic energy, and the internal energy per particle of the gas is $\frac{3}{2}k_B T$. The average rotational kinetic energy of the diatomic gas is $k_B T$ per particle, while the total internal energy is $\frac{3}{2}k_B T$ per particle.

c) For this class, we assume that the temperature is independent of time. The most obvious consequence is that the heat flow is not changing in time, so $P(r, t) = P(r)$. Furthermore, if the temperature is constant in time the total heat into a slice dr of the Earth must be zero, i.e. $dP/dr = 0$. Thus the rate of heat flow is also independent of r.

d) Our general formula for the rate of heat flow is,

$$P(r, t) = -kA\frac{dT}{dx}$$

Specifically for a sphere, this becomes

$$P(r, t) = -k(4\pi r^2)\frac{dT}{dr}$$

As we argued in part (c), $P(r, t)$ is a constant so we can use separation of variables

$$dT \propto \frac{dr}{r^2}$$

which implies that $T(r) = \frac{A}{r} + B$ where A and B are determined by the boundary conditions. In this case, we need

$$T(R_c) = T_c = \frac{A}{R_c} + B$$

$$T(R_s) = T_s = \frac{A}{R_s} + B$$

Solving these equations,

$$A = \frac{R_s R_c}{R_s - R_c}(T_c - T_s)$$

$$B = \frac{R_s T_s - R_c T_c}{R_s - R_c}$$

Physics 7B Spring 2015 Section 1 Prof. Bordel

Trevor Grand Pre

February 25, 2015

1 Problem 3

a) Since the process from point a to b is isothermal, we can use the fact that $PV = const.$ to find the pressure at point b:

$$P_b = \frac{P_a V_a}{V_b}. \tag{1.1}$$

Since the process from point c to a is adiabatic we can use the fact that $PV^\gamma = const.$ to find the pressure at point c:

$$P_c = \frac{P_a V_a^\gamma}{V_b^\gamma}, \tag{1.2}$$

and the ideal gas law tells us that

$$T_c = \frac{T_a V_a^{\gamma-1}}{V_b^{\gamma-1}}. \tag{1.3}$$

b) The work done from a to b is

$$W_{ab} = \int_{V_a}^{V_b} P_a dV \tag{1.4}$$

$$= \int_{V_a}^{V_b} NKT_a \frac{dV}{V} \tag{1.5}$$

$$= NKT_a \ln\left(\frac{V_b}{V_a}\right) \tag{1.6}$$

$$= P_a V_a \ln\left(\frac{V_b}{V_a}\right). \tag{1.7}$$

There is no work done on the path of b to c because the volume is constant. The work for the adiabatic expansion from c to a is found from

the first law of thermodynamics which states that $dE = dQ - PdV$. Since no heat is released, $dE = -W$, the work is minus the change in internal energy.

$$
\begin{aligned}
W_{ca} &= -\frac{5}{2}NK(T_a - T_c) & (1.8) \\
&= -\frac{5}{2}NK(T_a - T_a(\frac{V_a}{V_b})^{\gamma-1}) & (1.9) \\
&= -\frac{5}{2}P_aV_a(1 - (\frac{V_a}{V_b})^{\gamma-1}), & (1.10)
\end{aligned}
$$

where the last two steps were made by substituting the value for T_c, found in equation 2, and by considering the ideal gas law.

Hence the total work done during the cycle is

$$
W_{net} = P_aV_a\ln(\frac{V_b}{V_a}) - \frac{5}{2}P_aV_a(1 - (\frac{V_a}{V_b})^{\gamma-1}) \qquad (1.11)
$$

c)Heat is going into the system only on the path c to b. This is because during an adiabatic process, c to a, there is no heat flow and the isothermal process, a to b, releases heat in order to maintain constant temperature during contraction. Since there is constant volume, no work is being done and the flow of heat is equal to the change in internal energy, $dE = dQ$. Along that path the heat going into the system is

$$
\begin{aligned}
Q_{cb} &= \triangle E & (1.12) \\
&= \frac{5}{2}NK((T_c - T_b) & (1.13) \\
&= \frac{5}{2}NK((T_c - T_a) & (1.14) \\
&= \frac{5}{2}NKT_a((\frac{V_a}{V_b})^{\gamma-1} - 1) & (1.15) \\
&= \frac{5}{2}P_aV_a((\frac{V_a}{V_b})^{\gamma-1} - 1) & (1.16)
\end{aligned}
$$

d)The efficiency of this engine is

$$e = \frac{W}{Q_{in}} \tag{1.17}$$

$$= \frac{P_a V_a \ln\left(\frac{V_b}{V_a}\right) - \frac{5}{2} P_a V_a (1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1})}{\frac{5}{2} P_a V_a (\left(\frac{V_a}{V_b}\right)^{\gamma-1} - 1)} \tag{1.18}$$

$$= \frac{\ln\left(\frac{V_b}{V_a}\right) - \frac{5}{2}(1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1})}{\frac{5}{2}(\left(\frac{V_a}{V_b}\right)^{\gamma-1} - 1)} \tag{1.19}$$

when we compare this to the Carnot engine, We see that

$$\frac{e_{carnot}}{e_{system}} = 1 - \frac{T_c}{T_a} \cdot \frac{\frac{5}{2}\left(\frac{V_a}{V_b}\right)^{\gamma-1} - 1}{\ln\left(\frac{V_b}{V_a}\right) - \frac{5}{2}(1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1})} \tag{1.20}$$

$$= 1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1} \cdot \frac{\frac{5}{2}(\left(\frac{V_a}{V_b}\right)^{\gamma-1} - 1)}{\ln\left(\frac{V_b}{V_a}\right) - \frac{5}{2}(1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1})} \tag{1.21}$$

$$= -\frac{\frac{5}{2}(\left(\frac{V_a}{V_b}\right)^{\gamma-1} - 1)^2}{\ln\left(\frac{V_b}{V_a}\right) - \frac{5}{2}(1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1})} \tag{1.22}$$

$$= \frac{1}{\frac{\frac{2}{5}\ln\left(\frac{V_b}{V_a}\right)}{(\left(\frac{V_a}{V_b}\right)^{\gamma-1} - 1)^2} + \frac{1}{(1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1})}} \tag{1.23}$$

2   Problem 4

a) The second law of thermodynamics tells us that $\Delta S \geq 0$. When two object are in contact the hotter object will spontaneously give heat to the colder object. However, The spontaneous flow of heat from cold to hot would create a negative change in entropy and so the two objects returning to their original temperatures is not possible. Another way to answer this is by noting that two objects cannot be in thermodynamic equilibrium if they have different temperatures so the process of heat exchange cannot be reversible.

b)By using the fact that $Q_{si} = -Q_{hg}$ we see that the final temperature is

$$T_F = \frac{m_{Hg} c_{Hg} T_{Hg} + m_{Si} c_{Si} T_{Si}}{m_{Hg} c_{Hg} + m_{Si} c_{Si}} \tag{2.1}$$

c) From $dQ = TdS$, we can see that the change in entropy is

3

$$dS = \int \frac{dQ}{T} \tag{2.2}$$

$$= m_{Si}c_{Si}\int \frac{dT}{T} + m_{Hg}c_{Hg}\int \frac{dT}{T} \tag{2.3}$$

$$= m_{Si}c_{Si}\ln\left(\frac{T_F}{T_{Si}}\right) + m_{Hg}c_{Hg}\ln\left(\frac{T_F}{T_{Hg}}\right) \tag{2.4}$$

d) The object that is loosing heat will decrease in entropy the object gaining the heat will increase in entropy. I expect the two objects in thermal contact to transfer heat in a way that will allow the overall entropy to increase. There is more "order" when there is a hotter object and a colder object than if the two objects were in thermal equilibrium.

a)

First we will calculate the electric field due to the ring assuming the center of the ring is at $x = 0$. (this is almost identical to the derivation on page 572 of the book). We will later shift the result to get the electric field due to the ring being at other positions.

Since there is radial symmetry all the components of the electric field tranverse to x from an element dq on the ring cancel. This is because at any point on the ring, the opposite point of the ring contributes the opposite way. Thus, we only need to find the electric field in the x direction:

The electric field is:

$$\vec{E} = \int \frac{dq}{4\pi\epsilon_0 s^2}\hat{s} = \int \frac{dqx}{4\pi\epsilon_0 s^3}\hat{x} + \int \frac{dqy}{4\pi\epsilon_0 s^3}\hat{y} + \int \frac{dqz}{4\pi\epsilon_0 s^3}\hat{z}$$

So the x-component is (if you didn't make a symmetry argument to select out the x component, you would see that the other integrals evaluate to 0):

$$E_x = \int \frac{dqx}{4\pi\epsilon_0 s^3}$$

Note that here, s is the distance from the element dq on the ring to the point where we want to find the electric field (where the point charge is). By geometry we get that:

$$s^2 = R^2 + x^2$$

We can rewrite $dq = \lambda dl$.

Now, the only thing that is changing in dl is the angle, since it is always at a fixed distance. Thus, noting that $l = R\theta$ along an arc, I get that $dl = Rd\theta$.

$$
\begin{aligned}
E_x &= \int \frac{\lambda dl x}{4\pi\epsilon_0 (R^2 + x^2)^{3/2}} \\
&= \int_0^{2\pi} \frac{\lambda R d\theta x}{4\pi\epsilon_0 (R^2 + x^2)^{3/2}} \\
&= \frac{\lambda 2\pi R x}{4\pi\epsilon_0 (R^2 + x^2)^{3/2}} \\
&= \frac{\lambda R x}{2\epsilon_0 (R^2 + x^2)^{3/2}}
\end{aligned}
$$

To get the electric field from a ring at a position that is not $x = 0$, you can make the replacement $x \to x - a$, which shifts to the *right* by a. The total electric field from both rings is, then, by superposition

$$E_x = \frac{\lambda R}{2\epsilon_0}\left(-\frac{x - l}{(R^2 + (x - l)^2)^{3/2}} + \frac{x + l}{(R^2 + (x + l)^2)^{3/2}}\right)$$

Plugging in $x = 0$ gives

$$E_x = \frac{\lambda}{2\epsilon_0}\left(\frac{l}{(R^2 + l^2)^{3/2}} + \frac{l}{(R^2 + l^2)^{3/2}}\right) = \frac{\lambda l}{\epsilon_0 (R^2 + l^2)^{3/2}}$$

b)

From part a)

$$E_x = \frac{\lambda R}{2\epsilon_0}\left(-\frac{x - l}{(R^2 + (x - l)^2)^{3/2}} + \frac{x + l}{(R^2 + (x + l)^2)^{3/2}}\right)$$

1

c)

Look at the limit as $x \to \infty$.

$$
\begin{aligned}
E_x &= \frac{\lambda R}{2\epsilon_0}\left(-(x-l)(R^2+x^2+l^2-2lx)^{-3/2}+(x+l)(R^2+x^2+l^2+2lx)^{-3/2}\right) \\
&= \frac{\lambda R}{2\epsilon_0}\left(-\frac{(x-l)}{x^3}\left(\frac{R^2}{x^2}+1+\frac{l^2}{x^2}-\frac{2l}{x}\right)^{-3/2}+\frac{(x+l)}{x^3}(\frac{R^2}{x^2}+1+\frac{l^2}{x^2}+\frac{2l}{x})^{-3/2}\right) \\
&\approx \frac{\lambda R}{2\epsilon_0}\left(-\frac{(x-l)}{x^3}\left(1-\frac{2l}{x}\right)^{-3/2}+\frac{(x+l)}{x^3}\left(1+\frac{2l}{x}\right)^{-3/2}\right) \\
&\approx \frac{\lambda R}{2\epsilon_0}\left(-\frac{(x-l)}{x^3}\left(1+\frac{3l}{x}\right)+\frac{(x+l)}{x^3}\left(1-\frac{3l}{x}\right)\right) \\
&\approx \frac{\lambda R}{2\epsilon_0}\left(-\frac{x-l}{x^3}+\frac{x+l}{x^3}-\frac{3l}{x^3}-\frac{3l}{x^3}\right) \\
&= -\frac{2\lambda R l}{\epsilon_0 x^3}
\end{aligned}
$$

In the second line I threw away the R^2 terms, since if $\frac{R}{x}$ is small, then $\frac{R^2}{x^2}$ is even smaller. In the third line I used the expansion $(1+z)^\alpha \approx 1+\alpha z$ when z is small. In the fourth line I again drop all R^2 terms. So we see that far away, the field scales like x^{-3}.

d)

Note that this plot has $l = R/2$ to make it plotable but the problem does not specify this