University of California, Berkeley,   Department of Physics   Spring Semester 2010

Physics 137A MIDTERM I EXAMINATION March 15, 2010 9:10-10:00 am

Advice:

Please cross out any work which you do not wish to be graded. If your paper is neat, clear, and easy to read, it could affect your grade favorably.

Partial credit will be given for an incomplete or incorrect solution only for relevant, applicable statements that are logically presented. Random, disconnected comments will not be credited even if they happen to be correct. If you are unable to complete the answer to a question, please state clearly how far you got, and indicate how you would proceed to a solution.

Information:

$\hbar \approx 10^{-34}$ joule sec = 0.66×10^{-15} eV sec
$c \approx 3 \times 10^{8}$ m sec^{-1}
1 eV = 1.6×10^{-19} joule
proton rest mass $\approx 1.7 \times 10^{-27}$ kg $\approx$ 1000 MeV/c^2
electron rest mass $m_0 \approx 9 \times 10^{-31}$ kg $\approx$ 0.5 MeV/c^2
Compton wavelength $\lambda_c = h/m_oc \approx 2.426$ pm
Boltzmann constant $k_B \approx 1.4 \times 10^{-23}$ JK^{-1}

1	9
2	7
3	9

25/30

9/10

1. (10 points)

In a Compton scattering experiment, x-rays with a wavelength of $\lambda = 0.1$ nm are scattered by electrons in a graphite target. The scattered photons are viewed at an angle of 90° (i.e. at right-angles) to the incident beam.

(a) Calculate the shift in wavelength $\Delta\lambda$ (including the sign).

(b) Calculate the energy imparted to a recoiling electron.

(c) Calculate the components of the momentum of the recoiling electron.

(d) After being hit by the photon, is the electron relativistic or non-relativistic? How do you know? [If you give a clear explanation you can get full credit even if your solution to part (b) is incorrect.]

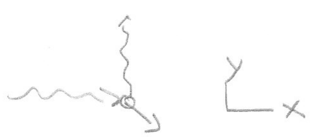
a. $\lambda_f - \lambda_i = -\frac{h}{m_0 c}(1-\cos\theta)$ $\qquad m_0 = m_e = 9\times10^{-31}$ kg

$\cos\theta = \cos(90) = 0$

$\lambda_f - \lambda_i = -\frac{h}{m_0 c} = \boxed{-2.454\times10^{-12}\,m}$ $\qquad$ 2/2 $\qquad \lambda_f = 9.975\times10^{-11}$ $\qquad \lambda_i = .1\times10^{-9}$

b. $\Delta E_\lambda = -\Delta E_{electron}$

$\Delta E_\lambda = \frac{hc}{\lambda_f} - \frac{hc}{\lambda_i} = 1.993\times10^{-15}$ $\quad 1.988\times10^{-15} - 4.78\times10^{-18}$ J



ΔE_{electr} $\boxed{4.78\times10^{-18}}$

2/2

c. in x direction,

$P_i = P_f$

$P_{\gamma i} + 0 = \cancel{0} + P_{ex}$ $\qquad P_{\gamma i} = P_{ex}$

$P_{\gamma i} = \frac{E}{c} = \frac{hc}{\lambda_i c} = \frac{h}{\lambda} = 6.626\times10^{-25}$ kgm/s

$\boxed{P_x = 6.626\times10^{-24}\,kgm/s}$ ✓ $\qquad$ 3/4

$P_{yf} = P_{yi}$

$0 = P_{\gamma f} + P_{ey}$ $\quad |P_{ey}| = |P_{\gamma f}| = \frac{h}{\lambda_f} = 6.628\times10^{-25}$ kgm/s

$\boxed{P_y = -6.623\times10^{-24}\,kgm/s}$ ✗

d. Rest energy of electron $= 9\times10^{-31}c^2 = 8.1\times10^{-14}$ J

ΔE (pt b) << E_{rest}

✓ so $\boxed{\text{non relativistic}}$

7/10

2. (10 points)

A body rotates about an axis for which its moment of inertia is I at an angular frequency ω. (Thus, the angular momentum is L = Iω, and the kinetic energy is $I\omega^2/2$.)

(a) Apply the Wilson-Sommerfeld quantization rule to find the possible values of its rotational energy E in terms of I and other applicable constants.

(b) Make an approximate numerical estimate for the lowest nonzero value of E for a hydrogen molecule rotating about an axis perpendicular to and at the midpoint of the line joining the two atoms. Assume the separation of the two atoms to be d = 0.1 nm. [The moment of inertia about an axis at the midpoint of the line joining the two H atoms and perpendicular to it is $2m(d/2)^2$.]

a. $\oint p\,dq = nh$

$\oint p\,dq = \oint L\,d\theta = \oint I\omega\,d\theta = I\omega(2\pi) = nh$

$L = n\hbar \qquad \omega = \frac{n\hbar}{I}$

5/6

$KE = I\omega^2/2 = \frac{I(n\hbar)^2}{2\,I^2}$

virial th: $K = -\frac{U}{2} = \frac{E}{2}$ dont need

$E = \cancel{2}\cdot\frac{n^2\hbar^2}{\cancel{2}\,I} = \boxed{\frac{n^2\hbar^2}{2I} \quad n = 1, 2, 3 \ldots}$

b. 

$E = \frac{n\hbar^2}{I} = \frac{n\hbar^2}{2m(d/2)^2} \qquad d = .1 \times 10^{-9}\,m,\ n = 1$

2/4

$= \boxed{2.22 \times 10^{-18}\,J}$

3. (10 points)

A particle of mass *m* undergoes one-dimensional simple harmonic motion along the x-axis. Its energy is $p^2/2m + \mu x^2/2$, where p is the x- momentum and μ is the spring constant.

Use the minimum uncertainty product to show that the lowest energy the particle can have is $\hbar\omega/2$, where you specify ω in terms of the given parameters. [If you are off by a numerical factor of order unity, don't worry!]

END

$E = \frac{p^2}{2m} + \frac{\mu x^2}{2}$ @min $\Delta x \Delta p = \hbar/2$ $\Delta p = \frac{\hbar}{2\Delta x}$

$E = \frac{\hbar^2}{2^2 \cdot 2m(\Delta x)^2} + \frac{\mu \Delta x^2}{2}$ $0 = \frac{dE}{d\Delta x} = \mu x + \frac{\hbar^2}{8m}\frac{(-2)}{x^3}$

$= \frac{\hbar^2}{8m\Delta x^2} + \frac{\mu \Delta x^2}{2}$

$\frac{\hbar^2}{4mx^3} = \mu x$ $x_{min} = \left(\frac{\hbar^2}{4m\mu}\right)^{1/4}$

$x_{min}^2 = \frac{\hbar}{2(m\mu)^{1/2}}$ ✓

$E = \frac{\hbar^2}{8mx^2} + \frac{\mu x^2}{2} = \frac{\hbar^2 \, 2\sqrt{m\mu}}{8m\hbar} + \frac{\mu\hbar}{2\sqrt{m\mu}} = \frac{\hbar}{4\sqrt{m}} + \frac{\mu\hbar}{2\sqrt{m}} = \frac{\hbar}{2}\left(\frac{1}{2\sqrt{m}} + \frac{1}{\sqrt{m}}\right)$

$E = \frac{\hbar^2}{8mx_m^2} + \frac{\mu x_m^2}{2} = \frac{\hbar^2 \, 2(m\mu)^{1/2}}{8m\hbar^2} + \frac{\hbar}{2 \cdot 2(m\mu)^{1/2}}\mu =$

$\frac{\hbar}{4m^{1/2}}\mu^{1/2} + \frac{\hbar \mu^{1/2}}{2 \cdot 2m^{1/2}} = \frac{\hbar}{2}\left(\left(\frac{\mu}{m}\right)^{1/2}\right) = \frac{\hbar}{2}\omega$ ✓

9/10