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137A Mid Term Exam, 60 minutes (Close Book)

1. (30%) We have a cubic cavity of dimensions $L \times L \times L$ filled with blackbody radiation. Find the ratio between the total blackbody energy contained in the box at temperature T and $2T$. The blackbody spectrum is

$$\rho(\omega) = \frac{\omega^2}{\pi^2 c^3} \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}.$$

(Hint to find the ratio you do not need to actually explicitly compute $\int_0^\infty \rho(\omega) d\omega$.)

2. (30 %) Consider a particle in a 1D box ($0 \leq x \leq L$). Recall the energy eigen functions are

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}.$$

Suppose at time zero the particle's wavefunction is

$$\psi(x, 0) = \frac{2}{\sqrt{L}} \sin \frac{2\pi x}{L} \cos \frac{\pi x}{L},$$

what is the wavefunction at time $t = \pi/(\hbar\pi^2/2mL^2)$. (You might find the following trigonometric formula useful: $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$.)

3. (40%) Use the operator method involving a and a^+ to compute the mean square of the position $< x^2 >$ in the first excited state of the simple harmonic oscillator. Recall that

$$a = \frac{1}{\sqrt{\hbar\omega}} \Big(\frac{p}{\sqrt{2m}} - i\sqrt{\frac{K}{2}} x \Big)$$

$$a^+ = \frac{1}{\sqrt{\hbar\omega}} \Big(\frac{p}{\sqrt{2m}} + i\sqrt{\frac{K}{2}} x \Big),$$

where K is the spring constant, m is the mass of the particle, and $\omega = \sqrt{K/m}$ is the oscillation frequency. (Hint: you need to first express x in terms of a and a^+.)

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