Physics 7B, Speliotopoulos
Second Midterm, Spring 2014
Berkeley, CA

Rules: *This midterm is closed book and closed notes. In particular, calculators are not allowed during this exam. Cell phones must be turned off during the exam, and placed in your backpacks, or bags.* ***They cannot be on your person.***

Please make sure that you do the following during the midterm:

- ***Show all your work in your blue book***
- *Write your name, discussion number, ID number on all documents you hand in.*
- *Make sure that the grader knows what s/he should grade by circling your final answer.*
- *Cross out any parts of your solutions that you do not want the grader to grade.*

Each problem is worth 20 points. We will give partial credit on this midterm*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

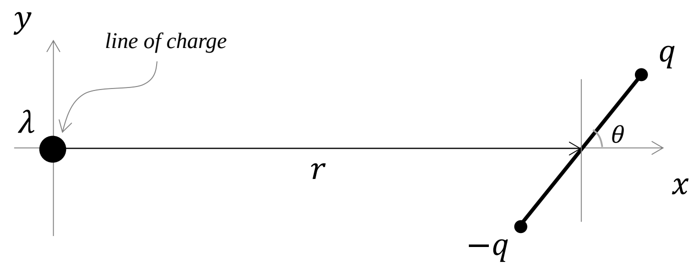
Copy and fill in the following information on the front of your bluebook:

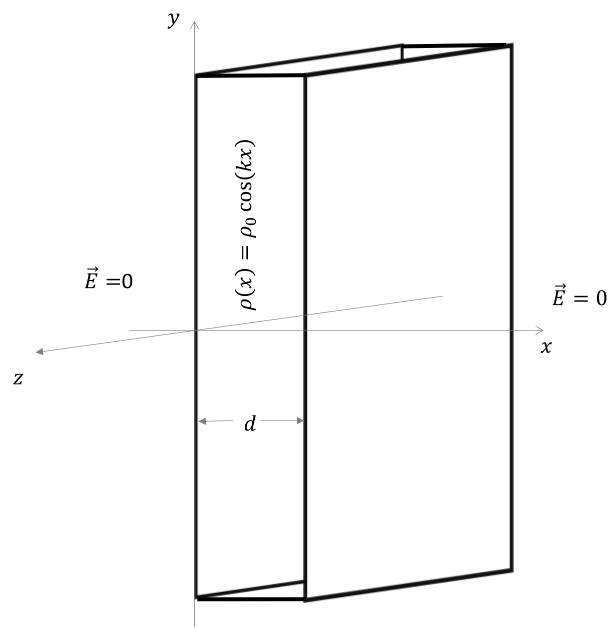
Name: ______________________________ *Disc Sec Number:* _______________________

Signature: ___________________________ *Disc Sec GSI:* __________________________

Student ID Number: ____________________

1. The figure to the right shows a two equal and opposite charges, q, separated by a fixed distance, d. The rod connecting the two charges is at an angle, θ, from the horizontal axis, and at a distance, $r \gg d$, from an infinite line of charge with charge per unit length, $\lambda > 0$. ***This line of charge points out of the page.*** Find the force, $\vec{F}$, on the charges to the lowest, nontrivial order in d/r when $\theta = 0$ and when $\theta = 90^{\circ}$. Express them in terms of any of the variables given and ϵ_0.

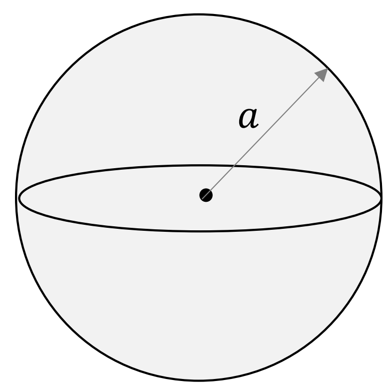




2. The figure to the right shows an infinite slab of charge that has a width, d, and within which the charge density,

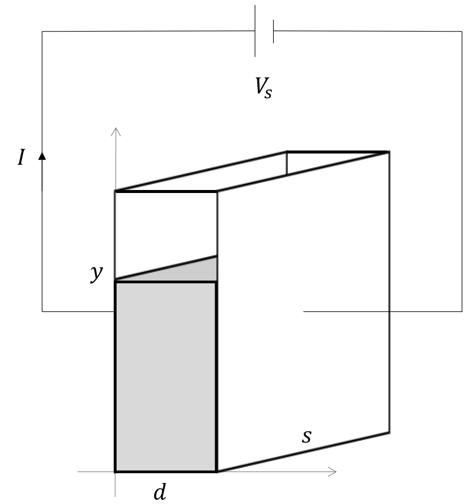
$$\rho(x) = \rho_0 \cos\left(\frac{n\pi x}{d}\right).$$

   a. The electric field is zero for $x \leq 0$ **and** $x \geq d$. What, then, are the possible values of n?
   b. Take $V(0) = 0$. Under the conditions in part a, what is the electric potential, $V(x)$, inside the slab (for $0 \leq x \leq d$)? Express it in terms of $n, \pi, \rho_0, \epsilon_0,$ d, and x.

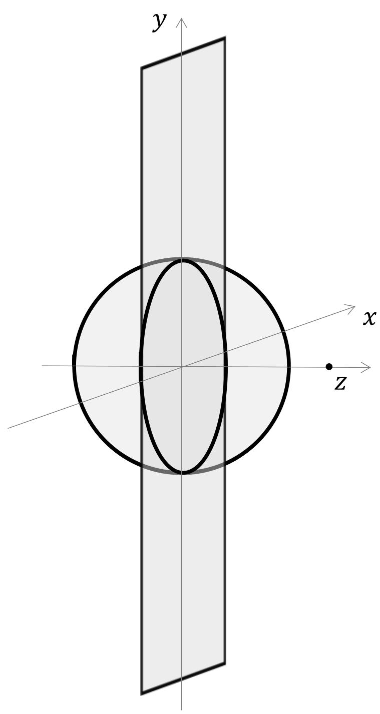


3. The figure to the right shows a charge, Q, surrounded by a spherical insulator with charge density, ρ, and radius, a. If the work, W, required to construct spherical charge distribution surrounding Q by bringing in those charges in from infinity is ***zero***, what is Q in terms of ρ, a, and π?

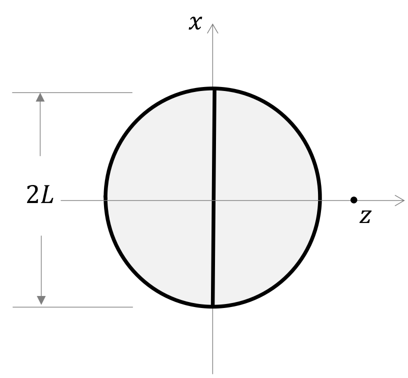
4. The figure to the right shows a partially filled container containing a fluid with a dielectric constant, K. The container consists of four sides made out of an insulator, and two opposing sides made out of metal. These two metal plates form a square parallel plate capacitor with sides, s, and a separation, d. This capacitor is connected to a voltage source, V_S, and is used to determine the rate at which the fluid is filling or emptying the container. The insulating sides of the container have negligible effects on the capacitance of the capacitor, and the rest of the container is filled with air for which $K_{air} \approx 1$.
   a. There is a current I from the capacitor to the voltage source (see figure). Is the fluid filling up the container, or emptying from it?
   b. What is the rate, $\frac{dV}{dt}$, at which the ***volume*** of the fluid in the container is changing? Express your answer in terms of any of the variables given and ϵ_0.



5. An infinitely long strip with constant charge density $\sigma_L > 0$ and width $2L$ cuts through the center of a spherical shell with diameter $2L$, and a constant charge density $\sigma_S < 0$ (see figure below). Both the strip and the sphere are insulators and have negligible thickness. What is the electric field, $\vec{E}(0,0,z)$ ***at any point***, z, along the z-axis shown in the figure? Express it in terms of any of the variables given and ϵ_0.



Perspective View



Top View

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} \text{ (point charge)}$$

$$d\vec{E} = \frac{d\lambda}{2\pi\epsilon_0 r}\hat{r} \text{ (line charge)}$$

$$\lambda = \frac{dQ}{ds} \qquad \sigma = \frac{dQ}{dA} \qquad \rho = \frac{dQ}{dV}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$dV = \frac{dQ}{4\pi\epsilon_0 r} \text{ (point charge)}$$

$$dV = -\frac{d\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{r_0}\right) \text{ (line charge)}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$C = \frac{\epsilon A}{d}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2} |\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\theta) \text{ (law of cosines)}$$