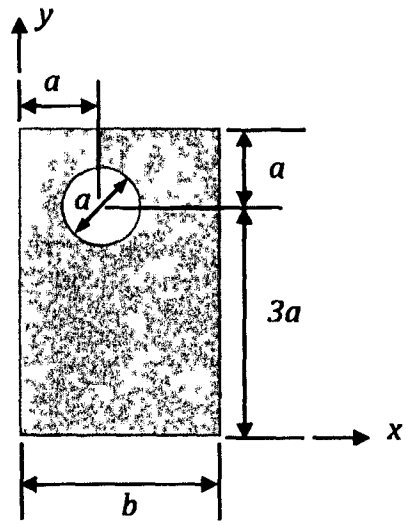
# Introduction to Solid Mechanics
# ME C85/CE C30

## Midterm Exam 2

## Fall, 2014

1. Do not open the exam until you are told to begin.
2. There is a separate answer booklet for each of the four problems of the exam. When you have finished the exam, please place each booklet in the appropriate place so that we can have them all separated once everyone is finished.
3. Put your name and SID on **every** page of your answer books.
4. You may not use a calculator, but you may use a straightedge to help you draw figures.
5. You may use one 8-1/2 x 11 sheet of notes, but not your book or any other notes.
6. Store everything else out of sight.
7. Turn off cell phones.
8. Please read the entire exam before starting work. You may solve the problems in any order you choose, of course, but pay attention to the clock so that you have sufficient time to work on all four problems.
9. There will be no questions during the exam. Write your concerns or alternative interpretations on your answer sheets.
10. Be concise and write clearly. Identify your answers by putting boxes around them.
11. You may leave the exam room when you are finished, but you may not leave and return during the exam. Please plan accordingly.
12. Time will be strictly enforced. At 9:00, you must put down your pencil or pen and immediately turn in your exam. Failure to do so may result in loss of points.

**Problem 1 (15 Points Total)** Determine the location of the centroid of the area shown consisting of a rectangle of area $4ab$, with a circular hole of diameter $a$ cut out. Express your answers in relation to the coordinate system provided, with its origin at the bottom left of the rectangle.

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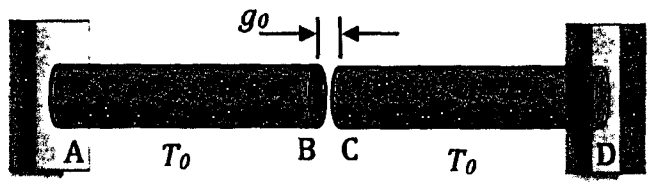
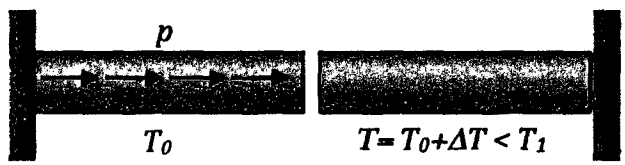
**Problem 2 (35 Points Total)** Consider two identical weightless coaxial rods AB and CD, with length $L$, area $A$, Young's modulus $E$ and linear coefficient of thermal expansion $\alpha$. Each is built into a rigid support at one end (A and D), and the rods are aligned such that the free ends (B and C) are initially separated by a small gap $g_0 << L$.

Rod AB is loaded by a uniform distributed axial force $p$ (with physical dimension of force per unit length). Rod AB is held at the ambient temperature $T_0$ throughout this problem. We take $p$ to be small enough that the deflection of end B is less than the initial gap ($\delta_B < g_0$).




Rod CD is subjected to a temperature increase of $\Delta T$ from the ambient temperature. No external forces are applied to CD.




Let $T_1$ be the temperature at which ends of the rods first touch due to the deflections of ends B and C, and let $\Delta T_1 = T_1 - T_0$.

a) At a temperature just below $T_1$, (just before the ends touch), determine the axial stresses in the rods at points A and D.

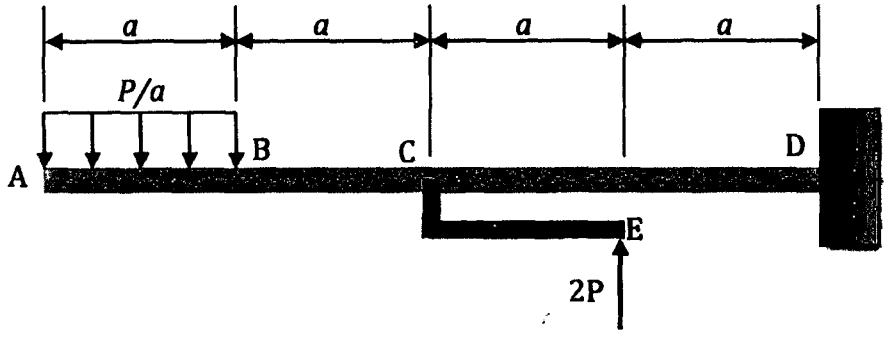
b) Derive an equation for $\Delta T_1$ in terms of the parameters given: $L$, $A$, $g_0$, $E$, $\alpha$, $p$ and $T_0$. (Note: Your equation may or may not involve all of these parameters.)

Now let the temperature increase to $T_2 > T_1$ so that the ends of the rods push against one another. As before, let $\Delta T_2 = T_2 - T_0$, but also let $\Delta T^*_2 = T_2 - T_1$.

c) At this increased temperature $T_2$, determine the **axial force** that the rods exert on one another. Express your answer in terms of the parameters given above: $L$, $A$, $g_0$, $E$, $\alpha$, $p$, $T_0$, $T_1$, $T_2$, $\Delta T_1$, $\Delta T_2$, and/or $\Delta T^*_2$. Your answer need not involve all of these parameters.

**Problem 3 (20 Points)** A uniform distributed load of magnitude $P/a$ is applied over the region from A to B of beam ABCD whose total length is $4a$. Another force of magnitude 2P is applied vertically at point E at the end of the rigid bracket CE. This bracket is rigidly attached to the beam at point C. The right end of the beam is built into a support at D.

Draw the shear and bending moment diagrams for the beam ABCD. Note that the bracket CE should not be part of the beam depicted in the diagrams. On each diagram, be sure to identify the location of the horizontal axis (i.e., where V or M is zero), the values of V or M at the ends and at all relevant transition points, and the slopes of portions of the diagrams where V or M vary linearly. For this problem, you will not be graded on whether you draw free body diagrams. It is up to you to assess their utility in drawing the shear and bending moment diagrams.

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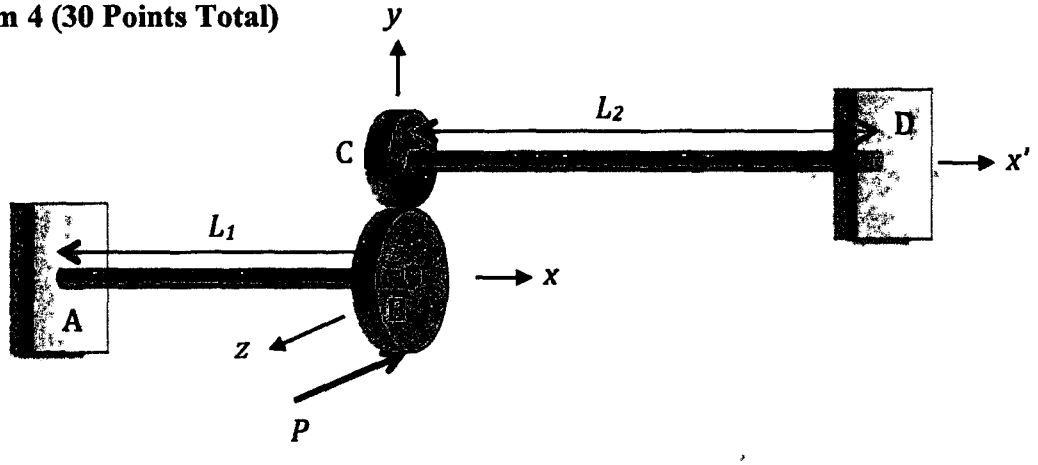
**Problem 4 (30 Points Total)**




A mechanism (not shown) below gear B exerts a force $P$ on that gear. Force $P$ is parallel to the z axis. Shaft AB is fixed at A and has length $L_1$; shaft CD is fixed at D and has length $L_2$. Both shafts have the same polar moment of inertia $J$ and are made of the same material with shear modulus $G$. Gear B has a radius of $r_B$ and gear C has a radius of $r_C$.

Determine the rotation of gear C under this loading. Your expression for the magnitude of the rotation should be in terms of the parameters given above: $P$, $L_1$, $L_2$, $r_B$, $r_C$, $J$ and $G$. Be sure to identify the direction of rotation as seen from the fixed end D.