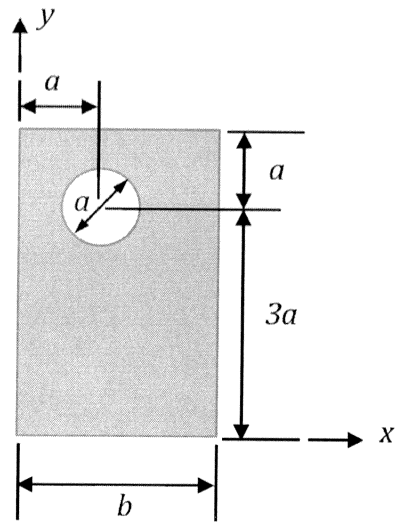
NAME SOLUTION SID ________

Problem 1



FOR THE RECTANGLE:

$$\left.\begin{aligned} A_R &= 4ab \\ \bar{x}_R &= b/2 \\ \bar{y}_R &= 2a \end{aligned}\right\} \Rightarrow$$

FIRST MOMENTS OF AREA

$$Q_{xR} = 8a^2 b$$
$$Q_{yR} = 2ab^2$$

FOR THE CIRCLE:

$$\left.\begin{aligned} A_c &= \pi a^2/4 \\ \bar{x}_c &= a \\ \bar{y}_c &= 3a \end{aligned}\right\} \Rightarrow$$

FIRST MOMENTS OF AREA

$$Q_{xc} = 3\pi a^3/4$$
$$Q_{yc} = \pi a^3/4$$

FOR THE COMPOSITE BODY:

$$\bar{y} = \frac{\sum Q_x}{\sum A} = \frac{\cancel{A_R}\, Q_{xR} - Q_{xc}}{A_R - A_c} = \frac{8a^2 b - 3\pi a^3/4}{4ab - \pi a^2/4} = \underline{\underline{\frac{32ab - 3\pi a^2}{16b - \pi a}}}$$

$$\bar{x} = \frac{\sum Q_y}{\sum A} = \frac{Q_{yR} - Q_{yc}}{A_R - A_c} = \frac{2ab^2 - \pi a^3/4}{4ab - \pi a^2/4} = \underline{\underline{\frac{8b^2 - \pi a^2}{16b - \pi a}}}$$

SANITY CHECKS:

IF $b \to \infty$, $\bar{y} = 2a$, $\bar{x} = b/2$ (LIKE THERE'S NO CIRCLE)

IF $b \to 2a$, $\bar{x} = \dfrac{32a^2 - \pi a^2}{32a - \pi a} = a$ (SYMMETRIC ABOUT $\bar{y}$ AXIS, BUT ... NOT MUCH TO SAY ABOUT $\bar{y}$)

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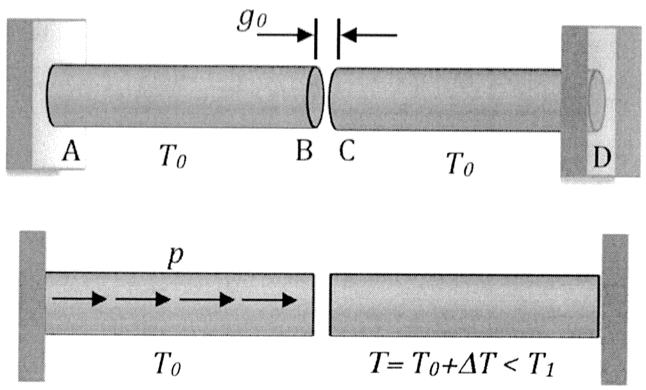
Problem 2

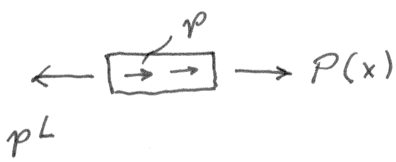


LET'S FOCUS FIRST ON AB:

FBD FROM 0 TO ~~x~~ L –

$$\Sigma F_x = 0 = \int_0^L p\,dx - R_A \Rightarrow \underline{\underline{R_A = pL}}$$

FBD FROM 0 TO $x < L$:



$$\Sigma F_x = 0 = -pL + \int_0^x p\,dx + P(x)$$
$$= -pL + px + P \Rightarrow \underline{\underline{P = p(L-x)}}$$

SINCE THE AXIAL FORCE IS A FUNCTION OF x, SO ARE THE AXIAL STRESS & STRAIN:

$$\sigma(x) = \frac{P(x)}{A} = \underline{\underline{\frac{p(L-x)}{A}}}, \quad \varepsilon(x) = \frac{\sigma(x)}{E} = \underline{\underline{\frac{p(L-x)}{AE}}}$$

THE DEFLECTION OF END B IS $\underline{\underline{\delta_B = \int_0^L \varepsilon\,dx = \frac{pL^2}{2AE}}}$

a) IN ROD AB, $\sigma(x) = \frac{p(L-x)}{A}$. AT $x=0$, $\boxed{\sigma_A = \frac{pL}{A}}$

IN ROD CD, $\varepsilon_T = \alpha\Delta T$ BUT SINCE END C IS NOT YET IN CONTACT WITH END B, $\boxed{\sigma = 0 \text{ EVERYWHERE IN CD}}$

NAME SOLUTION SID ____________

Problem 2

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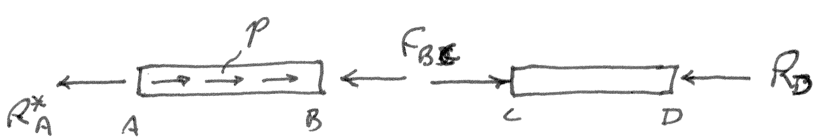
b) THE INITIAL GAP g_0 WILL BE CLOSED THROUGH A COMBINATION OF END B MOVING TO THE RIGHT BY AN AMOUNT δ_B AND END C MOVING TO THE LEFT BY AN AMOUNT δ_C. WE KNOW δ_B FROM BEFORE. δ_C IS THE TOTAL THERMAL EXPANSION OF CD.

$$\delta_{C_T} = \alpha \Delta T_1 L \Rightarrow \qquad g_0 = \delta_B + \delta_{C_T} = \frac{pL^2}{2AE} + \alpha \Delta T_1 L$$

SOLVING FOR ΔT_1 $\Rightarrow$ $\boxed{\Delta T_1 = \frac{1}{\alpha L}\left[g_0 - \frac{pL^2}{2AE}\right]}$

c) $T_2 > T_1$ IF C WAS NOT IMPEDED BY B, ROD CD WOULD EXPAND BY AN <u>ADDITIONAL</u> $\delta_{C_T}^* = \alpha \Delta T_2^* L$. SINCE C DOES HIT B AT THE TEMPERATURE T_1, RAISING THE TEMPERATURE TO T_2 CAUSES A FORCE TO BE EXERTED BY THE BARS ON EACH OTHER. HERE'S THE FBD'S OF THIS:



$$\Sigma F_x = 0 = pL - R_A^* - F_{BC} \qquad \Sigma F_x = 0 = F_{BC} - R_D^*$$

$$R_A^* = pL - F_{BC} \qquad R_D^* = F_{BC}$$

NAME SOLUTION SID

Problem 2

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THE MECHANICAL DEFORMATIONS DUE TO F_{BC} MUST BALANCE THE THERMAL DEFORMATION δ_c^*.

NOTE THAT WE HAVE ALREADY ACCOUNTED FOR THE DEFORMATIONS DUE TO p AND ΔT_1. WE ARE NOW FOCUSING ONLY ON THE TEMPERATURE CHANGE FROM T_1 TO T_2 ($\Delta T_2^* = T_2 - T_1$).

THE DEFLECTION OF END B DUE TO F_{AB} IS $\delta_B^* = \dfrac{F_{AB} L}{AE}$.

THE DEFLECTION OF END C DUE TO F_{AB} IS $\delta_c^* = \dfrac{F_{AB} L}{AE}$.

THE SUM OF THESE TWO MUST EQUAL $\delta_{c_T}^*$:

$$\frac{2 F_{AB} L}{AE} = \alpha \Delta T_2^* L \Rightarrow \boxed{F_{AB} = \frac{1}{2} AE \alpha \Delta T_2^*}$$

ALTERNATE FORMS:

$$\Delta T_2^* = T_2 - T_1 = \Delta T_2 - \Delta T_1 = \Delta T_2 - \frac{1}{\alpha L}\left[g_0 - \frac{pL^2}{2AE}\right] \Rightarrow$$

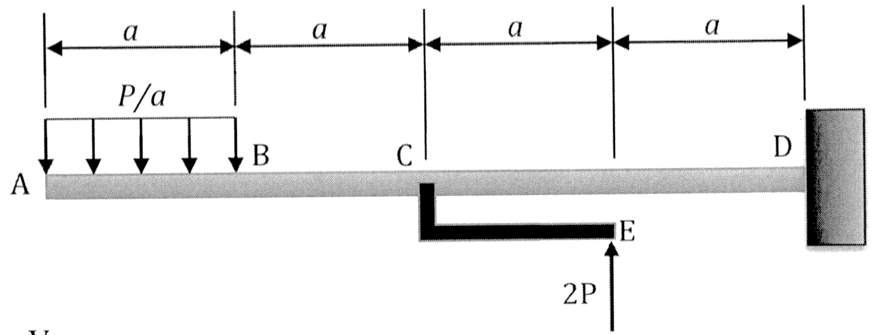
$$\boxed{F_{AB} = \frac{1}{2} AE\alpha\left[\Delta T_2 - \frac{1}{\alpha L}\left(g_0 - \frac{pL^2}{2AE}\right)\right]}$$

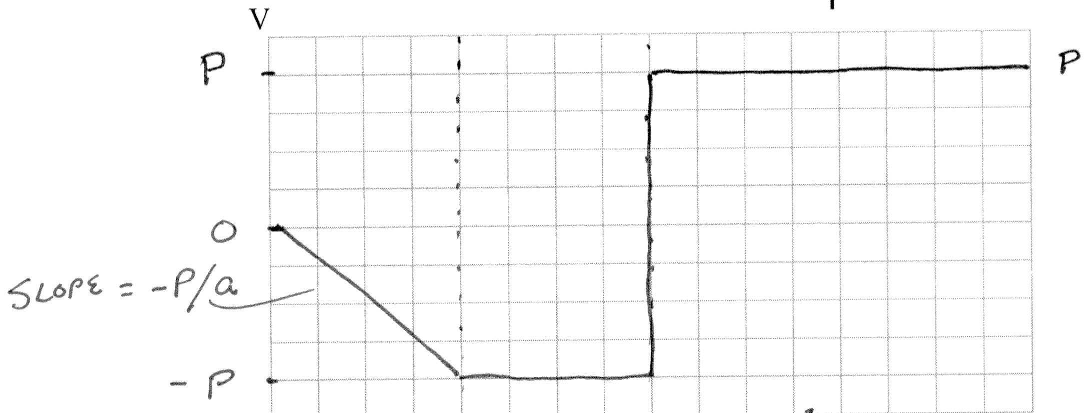
$$\Delta T_2 = T_2 - T_0 \Rightarrow$$

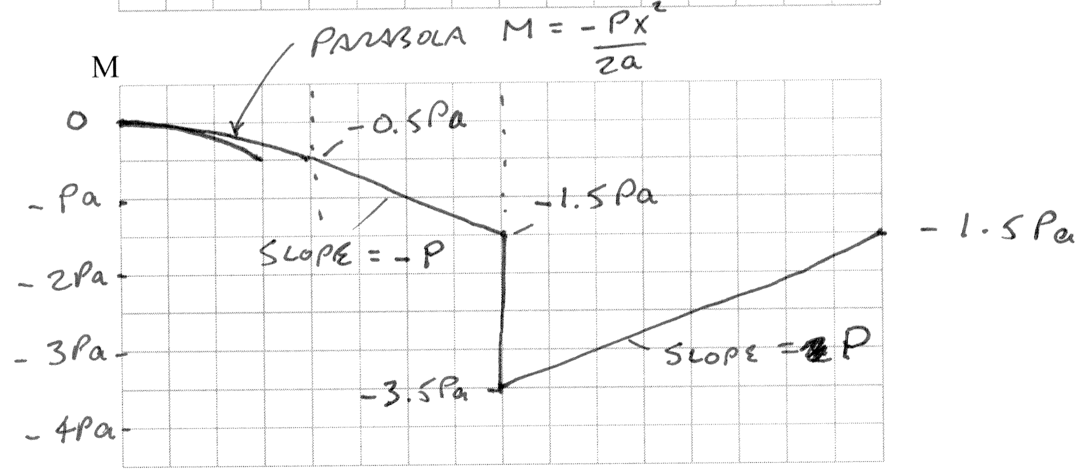
$$F_{AB} = \frac{1}{2} AE\alpha\left[T_2 - \underbrace{\left\{T_0 + \frac{1}{\alpha L}\left(g_0 - \frac{pL^2}{2AE}\right)\right\}}_{T_1}\right]$$

NAME SOLUTION SID ________________

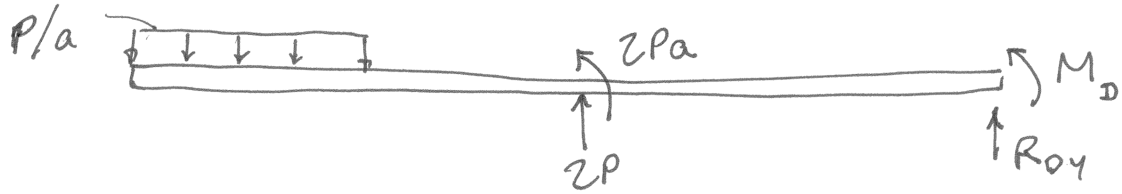
Problem 3







WE BEGIN BY REPLACING THE FORCE AT E WITH A STATICALLY EQUIVALENT FORCE - COUPLE MOMENT SYSTEM AT C. THE FBD OF THE BEAM IS THEN



NAME SOLUTION SID ______________

Problem 3

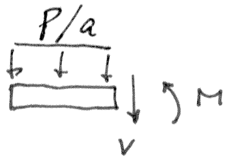
(PAGE 2)

SOLVE FOR R_{Dy} + M_D BY EQUILIBRIUM:

$$\sum F_y = 0 = -P + 2P + R_{Dy} \Rightarrow \underline{\underline{R_{Dy} = -P}}$$

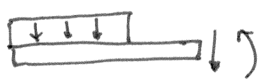
$$\sum M_D = 0 = \frac{7}{2}Pa - 2P(2a) + 2Pa + M_D \Rightarrow \underline{\underline{M_D = -\frac{3}{2}Pa}}$$

FBD A-B:



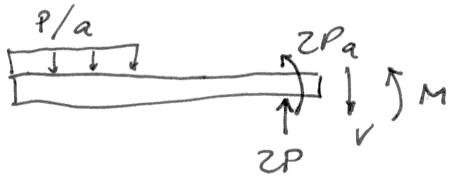
$$\Rightarrow V = -\frac{Px}{a}, \quad M = -\frac{Px^2}{2a}$$

FBD A-C:



$$\Rightarrow V = -P, \quad M = -\frac{Pa}{2} - P(x-a)$$

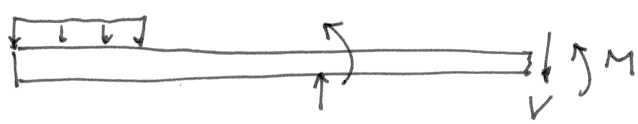
JUMPS AT C:



$$\Rightarrow V = P,$$

$$M = -\frac{3}{2}Pa - 2Pa$$

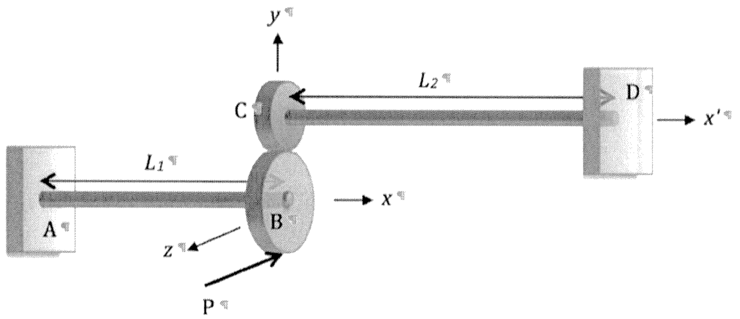
FBD A-D



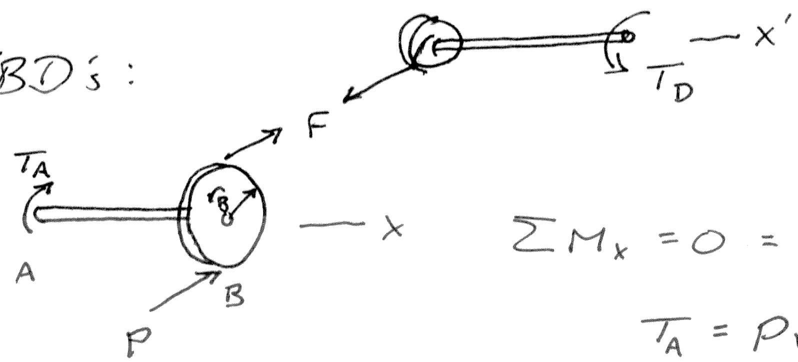
$$V = P, \quad M = -\frac{7}{2}Pa + P(x-2a)$$

NAME SOLUTION SID ______________

Problem 4



FBD's :



$$\Sigma M_{x'} = 0 = T_D - F r_c$$

$$T_D = F r_c \qquad (1)$$

$$\Sigma M_x = 0 = P r_B - F r_B - T_A \Rightarrow$$

$$T_A = P r_B - F r_B \qquad (2)$$

EQ's (1) + (2) INVOLVE 3 UNKNOWNS SO THE PROBLEM IS STATICALLY INDETERMINATE. WE NEED A COMPATIBILITY CONDITION. LET'S DETERMINE THE TWIST ANGLE OF B RELATIVE TO A AND D. THESE ANGLES MUST BE EQUAL.

FROM D: $\phi_{C1} = \dfrac{-T_D L_2}{GJ}$ (CW), $\phi_{B1} = -\dfrac{r_c}{r_B}\phi_{C1}$ (CCW)

FROM A: $\phi_{B2} = \dfrac{T_A L_1}{GJ}$

$$\phi_{B1} = \phi_{B2} = \frac{r_c}{r_B}\frac{T_D L_2}{GJ} = \frac{T_A L_1}{GJ} \Rightarrow T_A = \frac{r_c}{r_B}\frac{L_2}{L_1} T_D \qquad (3)$$

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Problem 4

(PAGE 2)

WE NOW HAVE 3 EQUATIONS IN THE 3 UNKNOWNS T_A, F & T_D:

$$T_D = F r_c \quad \Rightarrow \quad F = T_D / r_c$$

$$\left.\begin{aligned} T_A &= P r_B - F r_B \\ T_A &= \frac{r_c}{r_B}\frac{L_2}{L_1} T_D \end{aligned}\right\} \Rightarrow F = P - \left(\frac{r_c}{r_B}\right)\left(\frac{L_2}{L_1}\right) T_D$$

$$\Rightarrow \quad \frac{T_D}{r_c} = P - \left(\frac{r_c}{r_B}\right)\left(\frac{L_2}{L_1}\right) T_D \Rightarrow P = \left[\frac{r_B^2 L_1 + r_c^2 L_2}{r_B^2 r_c L_1}\right] T_D$$

$$\underline{\underline{T_D = \frac{r_B^2 r_c L_1}{r_B^2 L_1 + r_c^2 L_2} P}}$$

$$\boxed{\phi_c = \frac{-T_D L_2}{GJ} = \frac{-r_B^2 r_c L_1 L_2 P}{r_B^2 L_1 + r_c^2 L_2}} \quad (CW)$$

SANITY CHECKS: $\phi_c \to 0$ AS EITHER L_1 OR $L_2 \to 0$ ✓

LET $L_1 \to \infty$ (NO RESISTANCE FROM AB): $\phi_c = -\frac{(P r_c) L_2}{GJ}$ ✓

$T_A \to 0$, $F \to P$

LET $L_2 \to \infty$ (NO RESISTANCE FROM SHAFT CD): $\phi_c = -\left(\frac{r_B}{r_c}\right)\frac{(P r_B) L_1}{GJ}$ ✓

$T_D \to 0$, $F \to 0$