Physics 7B, Speliotopoulos
First Midterm, Spring 2014
Berkeley, CA

Rules: *This midterm is closed book and closed notes. You are allowed only the use of non-graphing scientific calculators, but not ones which can communicate with other calculators through any means.* ***Anyone who does use a wireless-capable device will automatically receive a zero for this midterm.*** *Cell phones must be turned off during the exam, and placed in your backpacks.* ***In particular, cell-phone-based calculators cannot be used.***

Please make sure that you do the following during the midterm:

- ***Show all your work in your blue book***
- *Write your name, discussion number, ID number on all documents you hand in.*
- *Make sure that the grader knows what s/he should grade by circling your final answer.*
- *Cross out any parts of your solutions that you do not want the grader to grade.*

Each problem is worth 20 points. We will give partial credit on this midterm*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

Copy and fill in the following information on the front of your bluebook:

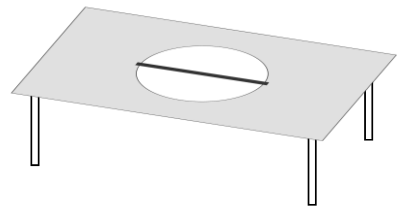
Name: ______________________________ *Disc Sec Number:* ________________________

Signature: ____________________________ *Disc Sec GSI:* ___________________________

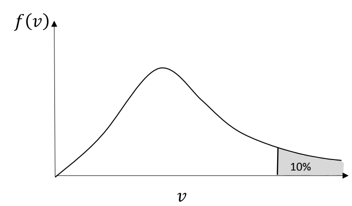
Student ID Number: ____________________

To maximize the credit you can received on each problem, do your calculations in terms of variables as much as possible before plugging in numbers.

1. A rod made out of aluminum ($\alpha_{Al} = 25 \text{ x } 10^{-6}\ °\text{C}^{-1}$) has a length, $L = 100$ cm, at temperature $T_0 = 100$ °C. It rests on top of a circular hole in a steel plate ($\alpha_{Fe} = 12 \text{ x } 10^{-6}\ °\text{C}^{-1}$) the forms the top of a table. This hole has a radius, $R = 49.7$ cm, at $T_0 = 20$ °C (see figure to the right). The center of the rod lies at the center of the hole. The temperature of the system is then changed to T. Is there a T below which the rod will fall through the hole? *This is not a qualitative question. You need to give a quantitative argument using the given numerical parameters to substantiate your answer.* Neglect the thickness of the rod, and the friction between the rod and the plate.

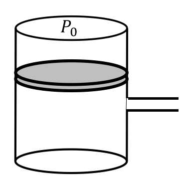


2. At extremely low temperatures, the usual methods of cooling a gas do not work. Instead, a form of evaporative cooling is used instead, and it is based on the fact that for a Maxwell speed distribution the top $p_N = 10\%$ of the molecules in a gas carry $p_E = 28\%$ of the total energy of the gas (see figure to the right). By removing these atoms and waiting for the gas to come back to equilibrium, you can cool the gas a fixed amount in each evaporation cycle.

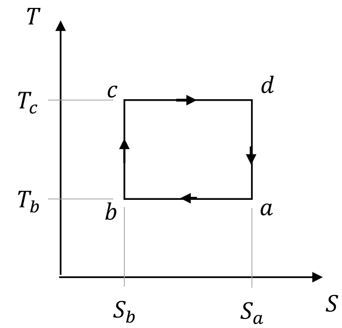


   a. Suppose at a gas has initial temperature, T_0, and number of particles, N. What is the total energy, E_1, of the system after the most energetic p_N percentage of particles are removed? Express your answer in terms of T_0, N, p_N, and k_B?
   b. After the system returns to equilibrium, what is the temperature, T_1, of the gas in terms of T_0? Use the p_N and p_E, given above. This is the result of one evaporation cycle.
   c. Suppose you wanted to cool the gas to below $T_0/2$. What is the minimum number, n, of cycles it will take?

3. The container to the right initially has a volume, V_0, and the *diatomic* gas within it has temperature, T_g. The piston at the top of the container has negligible mass, and can move up and down freely. The pressure outside of the container is at one atmosphere, P_0. Then n_V moles of water vapor (which have a mass m_V) with a temperature, T_V is injected quasi-statically into the container (see figure to right). After the system returns to equilibrium, water droplets cover the inside of the container, and the amount of water vapor remaining in the container is negligible. What is the volume, V, of the gas in the container now? *Water vapor has a degree of freedom, $d = 6$.* Express your answer in terms of any or all of the following: the variables V_0, P_0; n_V, m_v, T_V; the gas constant, R; the specific heats, $c_{water}$and c_{ice},; and the latent heat of evaporation, L_{vapor}, of water.

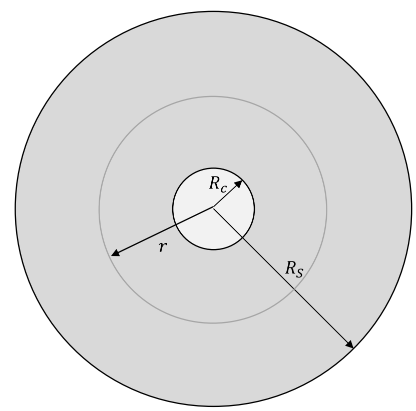


4. While we have been using *P-V* diagrams to describe the work done in thermodynamic cycles, temperature-entropy (*T-S*) diagrams are more useful to describe the flow of heat in the cycle. In a *T-S* diagram the temperature, T, of a thermal process is plotted as a function of the entropy, S, of it process, with the area under the graph giving the heat flow for the process. The figure on the right shows a thermodynamic cycle made up of four thermal processes.



   a. Determine whether each of the four processes, $a - b, b - c, c - d,$ and $d - a$, is isobaric, isovolumetric, isothermal, or adiabatic. Give reasons for your conclusions.
   b. What is the work done in the process? Calculate it using the *T-S* diagram to the right, and express this in terms of any or all of the variables given in the figure.
   c. Calculate the efficiency of the cycle directly using the *T-S* diagram, and express this in terms of any or all of the variables given in the figure.

5. A very crude and overly simplistic model for determining the temperature, $T(r)$, inside the Earth as a function of radius, r, is to consider it to consist of an inner, spherical core which has a radius, $R_c = 1220$ km that is at a fixed temperature, $T_c = 5430$ ℃ (see figure to right). This inner core is surrounded by solid basalt rock that has a thermal conductivity, $k = 1.70$ W/m·K, with the temperature of the Earth right below the surface at $R_S = 6380$ km, fixed at $T_S = 13$ ℃.



   a. Using physical principles and concepts, explain why the rate of heat flow, $\frac{dQ}{dt}$, from the inner core to the surface of the Earth is a constant.
   b. What is the temperature profile of the Earth, $T(r)$, in terms of $\frac{dQ}{dt}$, and the *variables* r, R_c, T_c, and R_S,
   c. What is $\frac{dQ}{dt}$? Calculate a numerical answer.

(In reality, the material between the inner core and the surface of the Earth is not solid, but instead a viscous liquid, and the dominant form of heat conduction is convection, not conduction. For this reason, your answer in part a is off by a factor of 200 or so.)

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} kT$$

$$f_{Maxwell}(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$E_{int} = \frac{d}{2} NkT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$\Delta E_{int} = Q - W$$

$$dE_{int} = dQ - PdV$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For a reversible adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2} R$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0 \text{ (For irreversible processes)}$$

$$\overline{g(v)} = \int_0^\infty g(v)\frac{f(v)}{N} dv \ \ (f(v) \text{ a speed distribution})$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2} x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

	Q	W
Isobaric	$C_P n\Delta T$	$P\Delta V$
Isochoric	$C_V n\Delta T$	0
Isothermal	$nRT\ln\left(\frac{V_f}{V_0}\right)$	$nRT\ln\left(\frac{V_f}{V_0}\right)$
Adiabatic	0	$-\frac{d}{2}(P_f V_f - P_0 V_0)$