PHYSICS 8B, FALL 2014

Lecture 1, Midterm 1

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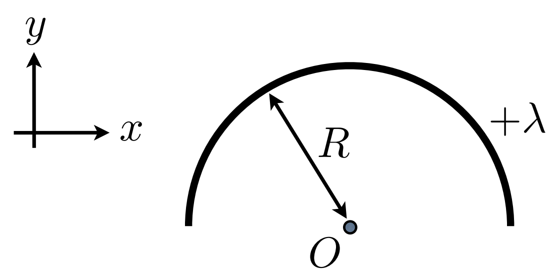
Wednesday, October 8th, 7pm-9pm

Make sure you show all your work and justify your answers in order to get full credit.

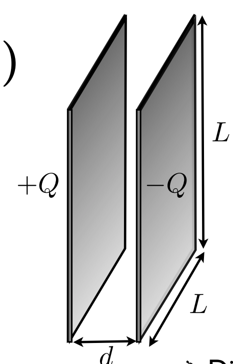
Problem 1 – Semicircular charged ring (25pts)

Calculate the electric field created at the center O of a ***uniformly*** charged semicircular ring of radius R. The linear charge density λ is positive. Remember to give both the magnitude and direction of the electric field.



Problem 2 – Parallel plate capacitor (25pts)

A capacitor is made of two parallel ***square*** plates of lateral size L, separated by a distance d, and carrying the charges $+Q$ and Q respectively. The space between the plates ***initially*** contains a vacuum.

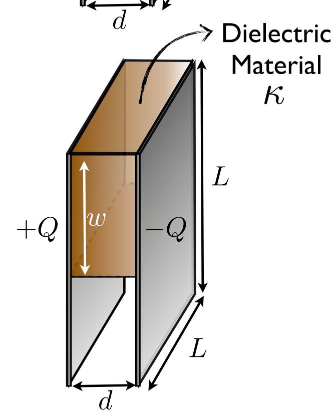


(a) Specify under which conditions those plates could be considered as uniformly charged infinite sheets, and determine the electric field inside and outside the capacitor.

(b) Express the voltage across the capacitors plates.

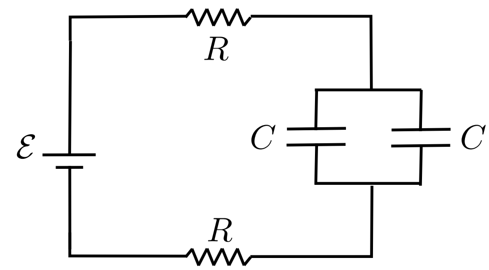
(c) Derive the capacitance C_0 of the parallel plate capacitor.

(d) Determine the capacitance C of the new capacitor if the gap between the plates is filled with an insulator of dielectric constant κ over the entire thickness d, length L and width $w < L$. *Hint: You may view this as a combination of capacitors.*



(Problem 2.d.)

Problem 3 – RC circuit (25pts)



In the right figure R and C are respectively a resistance and a capacitance, and $\mathcal{E}$ is the voltage sourced by the battery.

(a) Draw a simplified version of that electrical circuit using only one resistor of equivalent resistance R_{eq} and one capacitor of equivalent capacitance C_{eq}. Express R_{eq} and C_{eq} as a function of R and C.

(b) Before the battery is connected to the circuit, the capacitors are uncharged. Establish the differential equation satisfied by the charge Q accumulating on the equivalent capacitors plates, using R_{eq} and C_{eq}. *Hint: You don't need to solve the equation, the solution is provided on the equation sheet.*

(c)
1. Determine the current I going through the equivalent circuit immediately after the battery is connected to the circuit. *You may consider using the mathematical form of I(t) if you cannot think of any other way to answer the question.*
2. Sketch a qualitative plot of the current as a function of time.

(d) What is the maximal electric potential energy that can be stored by the equivalent capacitor, in terms of R_{eq} and C_{eq}?

Problem 4 – Resistivity and current (Conceptual questions) (25pts)

(a) A typical value of the magnitude of the drift velocity in a Cu wire is on the order of 0.1 mm/s, while the random speed is on the order of 10^6 m/s. Explain what causes the drift velocity to be so small.

(b) Based on the drift velocity reported in (a), it would take several hours for an electron to travel 1 m! Explain why the light goes on almost instantaneously when you flip a light switch. *Hint: remember what causes the electron motion.*

(c) If you stretch a wire of initial length ℓ_0 and cross-sectional area A_0 such that the length is doubled, how does the final resistance R compare to the initial resistance R_0, assuming that the resistivity ρ is unchanged in this process? Give a *qualitative* explanation for the *quantitative* result you got.

(d)
1. For a given voltage sourced by a battery, how does the power consumption change across the resistor if the resistance is doubled?
2. The resistor converts the electrical energy provided by the battery into another form of energy. Which form of energy is this?

Midterm 1 Equation Sheet

- $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^2}\,\hat{r}$
  $= k\frac{Q_1Q_2}{r^2}\,\hat{r}$
- $\vec{F} = Q\vec{E}$
- $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r^2}\,\hat{r}$
  $= k\int \frac{dQ}{r^2}\,\hat{r}$
- $\lambda = \frac{dQ}{d\ell}$
- $\sigma = \frac{dQ}{dA}$
- $\rho = \frac{dQ}{dV}$
- $\Phi_E = \int_{\mathcal{A}} \vec{E}\cdot d\vec{A}$
- $\oint_{\mathcal{A}} \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$
- $\vec{p} = Q\vec{d}$
- $\vec{\tau} = \vec{p}\times\vec{E}$
- $U = -\vec{p}\cdot\vec{E}$
- $\Delta U = Q\Delta V$
- $dV = -\vec{E}\cdot d\vec{\ell}$
- $V = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r}$
  $= k\int \frac{dQ}{r}$
- $Q = CV$
- $C_{eq} = C_1 + C_2$ (in parallel)
- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ (in series)
- $\epsilon = \kappa\epsilon_0$
- $U = \frac{Q^2}{2C}$
- $I = \frac{dQ}{dt}$
- $V = IR$
- $R = \rho\frac{\ell}{A}$
- $P = IV$
- $I = \int_{\mathcal{A}} \vec{j}\cdot d\vec{A}$
- $\vec{j} = nq\vec{v}_d = \frac{\vec{E}}{\rho}$
- $R_{eq} = R_1 + R_2$
  (in series)
- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
  (in parallel)
- $\sum_{\text{juntion}} I = 0$
- $\sum_{\text{loop}} V = 0$
- $Q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right)$
  (RC Circuit, charging)
- $Q(t) = C\mathcal{E}e^{-t/(RC)}$
  (RC Circuit, discharging)
- $\int x^m\,dx = \frac{x^{m+1}}{m+1}$ for $m \neq -1$
- $\int \frac{1}{x}\,dx = \ln x$

In the following, a is a constant:

- $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$
- $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
- $\int \frac{1}{\left(x^2+a^2\right)^{3/2}}\,dx = \frac{x}{a^2\sqrt{x^2+a^2}}$
- $\int \frac{x}{\left(x^2+a^2\right)^{3/2}}\,dx = -\frac{1}{\sqrt{x^2+a^2}}$
- $\int \sin x\,dx = -\cos x$
- $\int \cos x\,dx = \sin x$
- $\cos 0 = -\cos\pi = 1$
- $\sin\left(\frac{\pi}{2}\right) = -\sin\left(\frac{3\pi}{2}\right) = 1$
- $\cos\left(\frac{\pi}{2}\right) = \sin 0 = \sin\pi = 0$