PHYSICS 7B, Section 1 – Fall 2012
Final exam, C. Bordel
Monday, December 10, 2012
11:30 am – 2:30 pm

<u>Gradient operator and infinitesimal displacement in spherical coordinates</u> :

$$\vec{\nabla} f = \overrightarrow{grad}\, f = \left(\frac{\partial f}{\partial r}\right)\vec{u_r} + \left(\frac{1}{r}\frac{\partial f}{\partial \theta}\right)\vec{u_\theta} + \left(\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\right)\vec{u_\varphi}$$

$$\vec{dl} = dr\,\vec{u_r} + r d\theta\,\vec{u_\theta} + r\sin\theta d\varphi\,\vec{u_\varphi}$$

<u>Toolbox</u> :

$$L = \frac{N\phi}{I}\ ;\ M = \frac{N_2\phi_2}{I_1} = \frac{N_1\phi_1}{I_2}$$

$$\int\frac{dx}{1+x^2} = \arctan x\ ;\ \int\frac{dx}{(1+x^2)^{3/2}} = \frac{x}{\sqrt{1+x^2}}\ ;\ \int\frac{xdx}{1+x^2} = \frac{1}{2}Ln(1+x^2)$$

<u>Problem 1</u> – Thermodynamic processes (20 pts)

2 moles of an ideal gas with molar specific heat $C_v = \frac{5}{2}R$ are initially at temperature T_0 and pressure P_0. Vibrational degrees of freedom can be neglected in this temperature range.

a) How many degrees of freedom does the gas have? Could this be a monatomic gas?

Determine the change in internal energy ΔE, the temperature change ΔT and work W done by the gas when heat Q is added to the gas (b) isothermally, (c) isochorically, (d) isobarically.

<u>Problem 2</u> – Heat radiation of a lightbulb (10 pts)

The tungsten filament of a lightbulb has radius R and length l when it's hot (T).

a) Assuming the filament is an isolated system, express the radiative power (or rate of radiant heat flow) of the filament when it's hot.

b) Now assuming that the air surrounding the lightbulb is at room temperature (T_a), calculate the net radiative power of the lightbulb.

Problem 3 – **Model of the hydrogen atom (20 pts)**

The hydrogen atom can be modeled by considering a spherical negative charge distribution $\rho(r)$ around the proton of charge $q>0$. At distance r from the center O of the atom, the electric potential is given by the following expression :

$$V(r)=\frac{q}{4\pi\varepsilon_0 r}\exp\left(\frac{-r}{a}\right),$$ where a is a positive constant.

a) Determine the direction and magnitude of the electric field $\vec{E}(r)$ created by this charge distribution at a distance r from the origin.

b) Calculate the flux $\Phi_E(r)$ of the electric field through a sphere of center O and radius r.

c) Calculate the electric charge Q_i enclosed in the sphere of center O and radius r. What is the limit of Q_i when $r\to\infty$? Interpret your result.

d) Determine the negative charge distribution $\rho(r)$ using the infinitesimal charge dQ_i contained in a spherical shell of center O and inner and outer radii r and $r+dr$.

Problem 4 – **RC circuit (20 pts)**

Consider the following circuit (see Fig.1) where all the resistors have the same resistance R. At t=0, with the capacitor initially uncharged, the switch is closed.

a) At t=0 (right after the switch is closed), the 3 currents can be determined by analyzing a simpler but equivalent circuit. Draw the equivalent circuit and find I_1, I_2 and I_3 at t=0.

b) Same thing at $t=\infty$.

c) Determine the electric charge $Q(t)$ on the capacitor's plates assuming the form : $Q(t)=A+B\exp\left(\frac{-t}{\tau}\right)$.

d) When the capacitor is fully charged, the switch is reopened and a thin slab of glass, of dielectric constant K, is inserted between the plates of the capacitor. What is the voltage across the capacitor's plates in the presence of the dielectric? What is the time constant with which the capacitor will discharge? Draw the equivalent circuit.

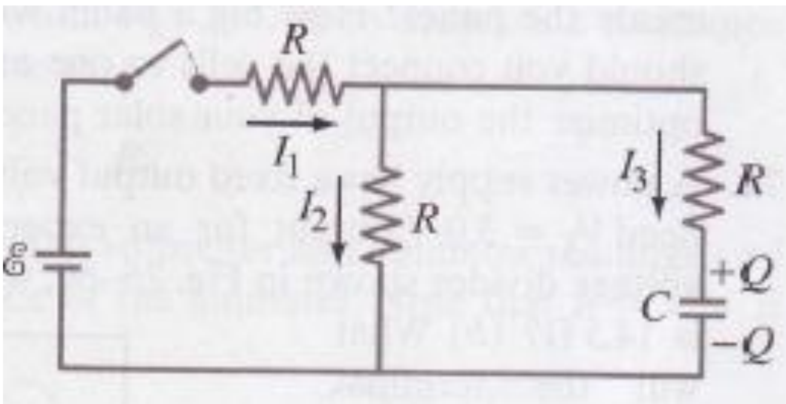

Figure 1

<u>Problem 5</u> – Hall effect (15 pts)

A long Cu strip of width w and thickness t is placed in a perpendicular magnetic field B. A longitudinal steady current I passes through the conductor, as shown in the following figure (Fig. 2).

a) Give the magnetic force $\vec{F}_B$ which acts on an electron moving with velocity $\vec{v}_d$ in the copper strip. Determine the Hall electric field $\vec{E}_H$ whose action on the electron would cancel the magnetic force.

b) Assuming all the electrons, of number-density n, are travelling at the same drift velocity $\vec{v}_d$ and experience zero net force, what is the potential difference or Hall voltage V_H between the sides of the copper strip?

c) What would be the effect on the measured voltage if the current I were unchanged, but generated by positive charge carriers ("holes") instead of electrons?

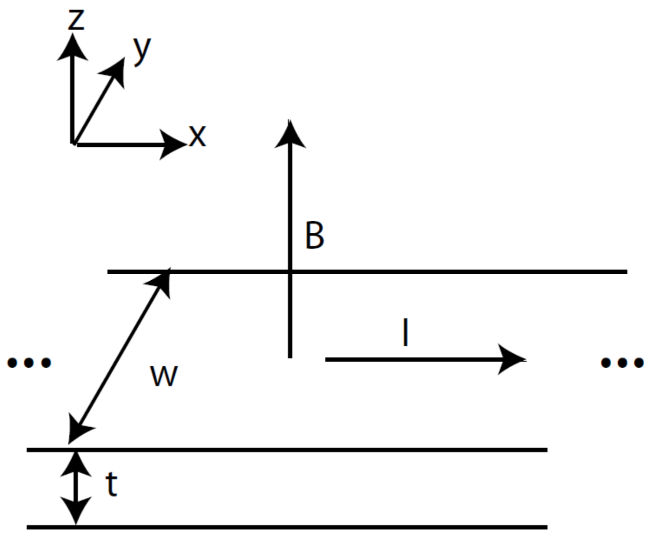

Figure 2

<u>Problem 6</u> – Electromagnetic induction (20 pts)

At t=0, a square coil of side $2d$ and resistance R is placed in a perpendicular time-dependant uniform magnetic field (see Fig. 3) of magnitude $B = B_0\ (1-kt)$, where k is a positive constant. No current initially passes through the coil.

a) Calculate the emf $\mathcal{E}$ induced in the loop.

b) Calculate the induced current i and explain its direction using Lenz's law.

c) Calculate the magnetic field B_i created by one side of the current-carrying loop at its center.

d) Calculate the total magnetic field $B_{tot}\ (t)$ at the center of the loop.

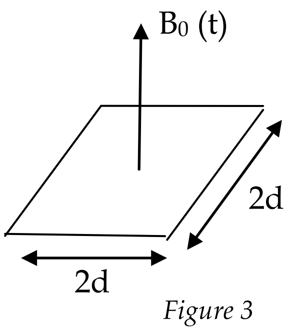

Figure 3

<u>Problem 7</u> – Inductors (20 pts)

A solenoid of length l and cross-sectional area A is made of N_p turns of a weakly resistive conducting wire.

a) Calculate the self-inductance L of the solenoid.
b) Calculate the energy U stored in this inductor when a current I passes through it.
c) If this solenoid is used as the primary coil of an ideal transformer (Fig. 4), what is the ratio of the currents I_p and I_s passing in the 2 coils?
d) What is the number of turns N_s required on the secondary coil in order to transform 110 V into 15 V? Use N_p =110.

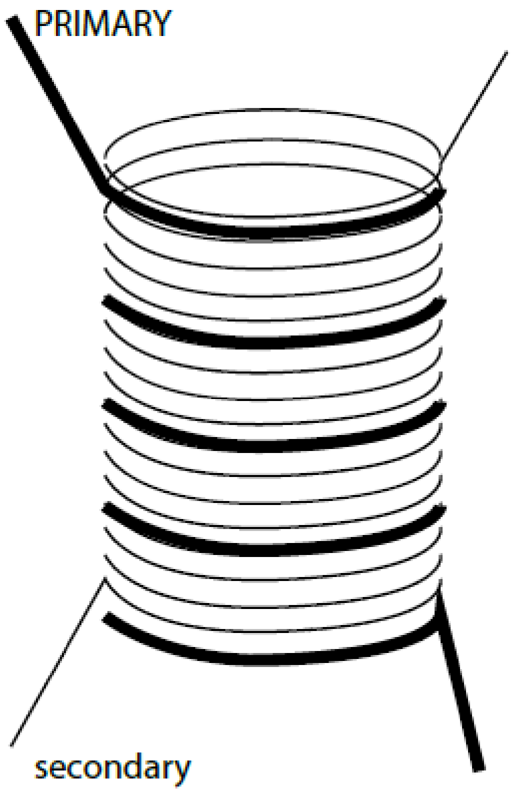

Figure 4