

MULTIPLE CHOICE SECTION, PROBLEMS 1-3 [2 POINTS EACH]
Please select one answer for all questions.

- (1) An Otto cycle with air as the working fluid has a compression ratio of 10. The engine receives 100 kW of heat from a high temperature reservoir at 3000 K and rejects 60 kW of heat to ambient air at 300 K. The entropy generation rate of this engine is

- a) 0 kW/K
b) 0.2 kW/K
c) 0.167 kW/K
d) 0.033 kW/K

$$\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} = \Delta \dot{S}_{sys} \quad \text{b/c steady state}$$

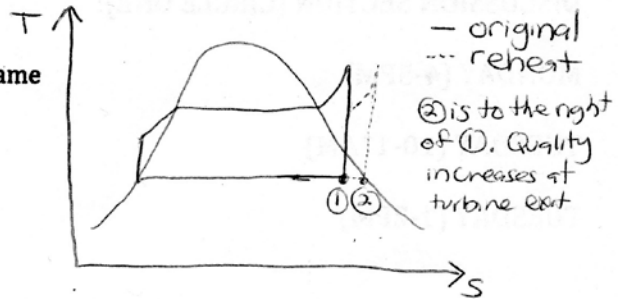
$$\left(\sum_{in} \frac{\dot{Q}_i}{T} + \sum_{in} \dot{m} s \right) - \left(\sum_{out} \frac{\dot{Q}_i}{T} + \sum_{out} \dot{m} s \right) + \dot{S}_{gen} = 0$$

$$\frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} = \frac{60 \text{ kW}}{300 \text{ K}} - \frac{100 \text{ kW}}{3000 \text{ K}} = 0.167 \text{ kW/K}$$

- (2) An ideal Rankine cycle operates at fixed boiler and condenser pressures. When the cycle is modified with reheating,

- a) the turbine output will decrease
b) the amount of heat reject will remain the same
c) the amount of heat input will decrease
d) the quality at turbine exit will increase



- (3) A cogeneration power plant produces 40 MW of net power, 50 MW of process heat, and rejects 60 MW of heat to the surroundings. What is the utilization factor of this plant?

- a) 50%
b) 60%
c) 70%
d) 80%

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{sys} \quad \text{b/c steady state}$$

$$\left(\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m} (h + ke + pe) \right) - \left(\dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m} (h + ke + pe) \right) = 0$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \dot{W}_{out} - \dot{W}_{in}$$

$$\dot{Q}_{in} - (\dot{Q}_{PH} + \dot{Q}_{reject}) = \dot{W}_{net}$$

$$\dot{Q}_{in} - (50 \text{ MW} + 60 \text{ MW}) = 40 \text{ MW}$$

$$\dot{Q}_{in} = 150 \text{ MW}$$

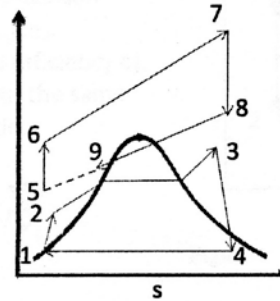
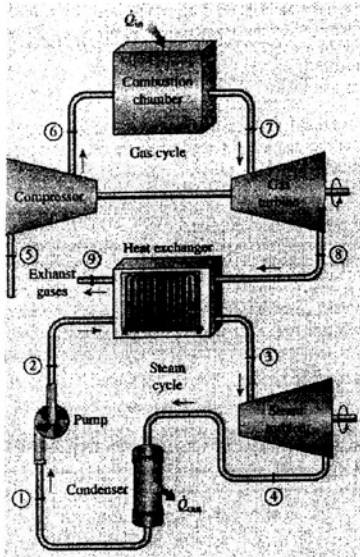
$$\epsilon_u = \frac{\dot{W}_{net} + \dot{Q}_{PH}}{\dot{Q}_{in}} = \frac{40 \text{ MW} + 50 \text{ MW}}{150 \text{ MW}}$$

$$\epsilon_u = 0.6$$

1-3	
4	
5	
Total	

(4) [12 points] Consider the gas-steam power plant shown in the figure below. The topping cycle is an ideal Brayton cycle with the temperature at each state point provided in the table below. Air enters the compressor at 10 kg/s. The bottoming cycle is a simple Rankine cycle operating between the pressure limits of 5 MPa and 10 kPa. Saturated liquid enters the pump. Steam is heated in a heat exchanger by the exhaust gases to a temperature of 500° C. The efficiency of the steam turbine is 85% and the efficiency of the pump is 90%. Assume air has a constant specific heat, $c_p = 1.05 \text{ kJ/kgK}$.

- Complete the table below with the enthalpies of each state point in the steam cycle.
- Find the thermal efficiency of the power plant.



Steam Cycle Enthalpies

State Point	Enthalpy (kJ/kg)
1	191.81
2	197.4
3	3434.7
4	2394.12

Gas Cycle Temperatures

State Point	Temperature
5	300 K
6	500 K
7	1200 K
8	750 K
9	400 K

Thermodynamic Properties of Water

Phase	Temperature	Pressure	Specific Volume (m ³ /kg)	Internal energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kgK)
Saturated Liquid	45.81 °C	10 kPa	0.001010	191.79	191.81	0.6492
Saturated Vapor	45.81 °C	10 kPa	14.670	2437.2	2583.9	8.1488
Superheated Gas	500 °C	5 MPa	0.06858	3091.8	3434.7	6.9781

a) State ①

$$P_1 = 10 \text{ kPa}$$

$$x_1 = 0$$

$$h_1 = 191.81 \text{ kJ/kg}$$

$$v_1 = 0.00101 \text{ m}^3/\text{kg}$$

State ②

$$w_{pump,s} = v_1 (P_2 - P_1)$$

$$w_{pump,s} = 0.00101 \text{ m}^3/\text{kg} (5,000 - 10) \text{ kPa}$$

$$w_{pump,s} = 5.04 \text{ kJ/kg}$$

$$\eta_p = \frac{w_{p,s}}{w_{p,a}}$$

$$0.9 = \frac{5.04}{w_{p,a}} \Rightarrow w_{p,a} = 5.6 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{p,a} = 197.4 \text{ kJ/kg}$$

State ③

$$T_3 = 500 \text{ C}$$

$$P_3 = 5 \text{ MPa}$$

$$h_3 = 3434.7 \text{ kJ/kg}$$

$$s_3 = 6.9781 \text{ kJ/kgK}$$

State ④

$$P_4 = 10 \text{ kPa}$$

$$s_{4s} = s_3 = 6.9781 \text{ kJ/kgK}$$

$$s_{4s} = s_f + x s_{fg}$$

$$6.9781 = 0.6492 + x (8.1488 - 0.6492)$$

$$x = 0.844$$

$$h_{4s} = 191.81 + 0.844 (2583.9 - 191.81)$$

$$h_{4s} = 2210.49$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$0.85 = \frac{3434.7 - h_4}{3434.7 - 2210.49} \Rightarrow h_4 = 2394.12 \text{ kJ/kg}$$

b) Find the mass flow rate of the steam (\dot{m}_s)
 Conservation of Energy of the Heat Exchanger

$$\dot{m}_s(h_3 - h_2) = \dot{m}_g(h_8 - h_9)$$

Air is an ideal gas

$$\dot{m}_s(h_3 - h_2) = \dot{m}_g c_p (T_8 - T_9)$$

$$\dot{m}_s = 1.135 \text{ kg/s}$$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

$$\dot{Q}_{in} = \dot{m}_g(h_7 - h_6) = \dot{m}_g c_p (T_7 - T_6)$$

$$\dot{Q}_{in} = 7350 \text{ kW}$$

$$\begin{aligned} \dot{W}_{net, steam} &= \dot{W}_{out, steam} - \dot{W}_{in, steam} \\ &= \dot{m}_s(h_3 - h_4) - \dot{m}_s w_{p, a} \\ &= 1174.7 \text{ kW} \end{aligned}$$

$$\begin{aligned} \dot{W}_{net, gas} &= \dot{W}_{out, gas} - \dot{W}_{in, gas} \\ &= \dot{m}_g(h_7 - h_8) - \dot{m}_g(h_6 - h_5) \\ &= \dot{m}_g c_p (T_7 - T_8) - \dot{m}_g c_p (T_6 - T_5) \\ &= 2625 \text{ kW} \end{aligned}$$

$$\dot{W}_{net} = \dot{W}_{net, steam} + \dot{W}_{net, gas}$$

$$\dot{W}_{net} = 3800 \text{ kW}$$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

$$\boxed{\eta_{th} = 51.7\%}$$

You may also solve this problem using

$$\eta_{th} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$

where

$$\dot{Q}_{out} = \dot{m}_g(h_9 - h_5) + \dot{m}_s(h_4 - h_1)$$

$$\dot{Q}_{out} = \dot{m}_g c_p (T_9 - T_5) + \dot{m}_s(h_4 - h_1)$$

$$\dot{Q}_{out} = 3549.6 \text{ kW}$$

The answer will be the same

(5) [12 points] The Atkinson engine is essentially an Otto four stroke engine with a different means of linking the piston to the crankshaft. The air-standard Atkinson cycle consists of isentropic compression, isochoric heat addition, isentropic expansion, and *isobaric* compression (See sketch below). At the beginning of the compression, $P_1 = 95 \text{ kPa}$ and $T_1 = 15^\circ \text{C}$. The compression ratio ($r = V_1/V_2$) is 9.5. The Atkinson ratio, which is defined as V_4/V_1 is 3. Assume constant specific heats: $c_p = 1.05 \text{ kJ/kgK}$ and $c_v = 0.763 \text{ kJ/kgK}$.

- (a) Find the thermal efficiency of the air-standard Atkinson cycle, $\eta_{AK} = 1 - q_{out}/q_{in}$
 (b) Find the thermal efficiency of an Otto cycle with the same compression ratio, $r = V_1/V_2 = 9.5$.

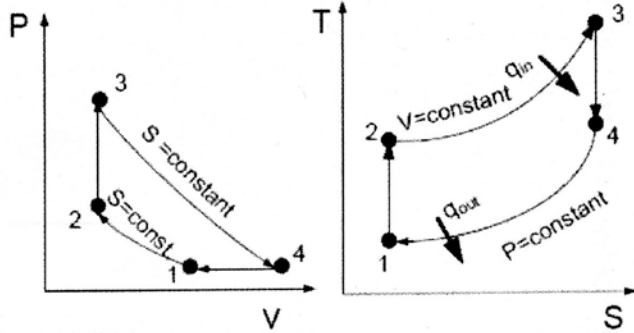


Figure 1: Sketch of P-V and T-S diagrams of Atkinson cycle

Let $A = \frac{V_4}{V_1}$

$k = \frac{c_p}{c_v} = \frac{1.05 \text{ kJ/kgK}}{0.763 \text{ kJ/kgK}} = 1.376$

a) $\eta_{AK} = 1 - \frac{q_{out}}{q_{in}}$

Heat enters between states (2)-(3)

$E_{in} - E_{out} = \Delta E_{sys}$
 $(Q_{in} + W_{in}) - (Q_{out} + W_{out}) = \Delta U + \Delta KE + \Delta PE$
 $Q_{in} = m \Delta u = m c_v \Delta T$
 $q_{in} = c_v (T_3 - T_2)$

Heat exits between states (4)-(1)

$E_{in} - E_{out} = \Delta E_{sys}$
 $(Q_{in} + W_{in}) - (Q_{out} + W_{out}) = \Delta U + \Delta KE + \Delta PE$
 $-Q_{out} - W_{out} = \Delta U$
 $-Q_{out} = \Delta U + W_{out}$
 Isobaric process: $\Delta H = \Delta U + W_{out}$
 $-Q_{out} = \Delta H$
 $Q_{out} = -\Delta H = -m(h_4 - h_1) = m(h_1 - h_4)$
 $Q_{out} = m c_p (T_4 - T_1)$
 $q_{out} = c_p (T_4 - T_1)$

$\eta_{AK} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p (T_4 - T_1)}{c_v (T_3 - T_2)}$

$\eta_{AK} = 1 - k \frac{T_4 - T_1}{T_3 - T_2}$

$\eta_{AK} = 66.6\%$

b) $\eta_{Otto} = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{9.5^{0.376}} = 0.571$

$\eta_{Otto} = 57.1\%$

State (1)

$T_1 = 15^\circ \text{C}$
 $T_1 = 288.15 \text{ K}$

State (2)

$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1}$

$T_2 = 672 \text{ K}$

State (4)

$P_1 = P_4$ $PV = mRT$
 $P = \frac{mRT}{V}$

$\frac{mRT_1}{V_1} = \frac{mRT_4}{V_4}$

$T_4 = T_1 \left(\frac{V_4}{V_1}\right) = T_1 A$

$T_4 = 864.45 \text{ K}$

State (3)

$\left(\frac{T_3}{T_4}\right) = \left(\frac{V_4}{V_3}\right)^{k-1}$

$\frac{V_4}{V_3} = \frac{V_4}{V_1} \frac{V_1}{V_3} = \frac{V_4}{V_1} \frac{V_1}{V_2} = Ar$

$T_3 = T_4 \left(\frac{V_4}{V_3}\right)^{k-1} = T_4 (Ar)^{k-1}$

$T_3 = 3,046.3 \text{ K}$