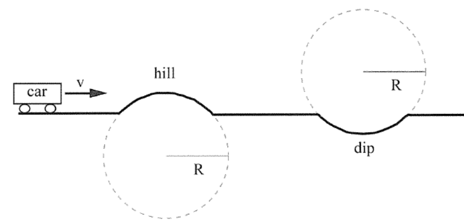
Physics 7A, Spring 2012, Section 3 Instructor: Professor Adrian Lee
Final Examination, Thursday, May 10, 2012

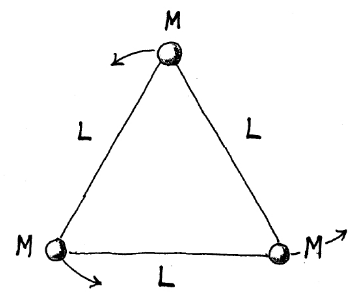
Please do work in your blue/greenbooks. Show your reasoning carefully so that we can be sure that you derived the answer rather than guessing it or relying on memory; in addition, this enables us to give partial credit. You may use three double-sided 3.5 x 5 index cards of notes. Test duration is three hours. Calculators are not allowed.

1 Hilly road [25 pts. total]



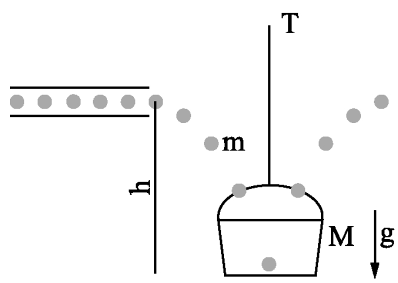
A car moves at a constant speed on a straight but hilly road. The car crosses over a hill and then a dip both of radius R.
a) As the car passes over the crest of the hill the normal force on the car is half the weight of the car. What is the normal force on the car as it passes over the center of the dip? Draw a free-body diagram for the car at the peak of the hill and bottom of dip. [15 pts]
b) What is the greatest speed the car can travel over the crest of the hill without leaving the road? [10 pts]

2 Orbits [25 pts. total]



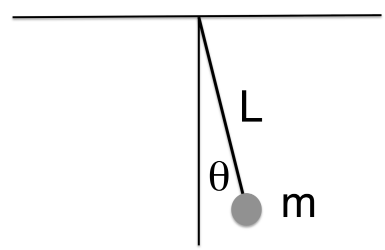
Three planets of identical mass M form the vertices of an equilateral triangle of side L and rotate in circular orbits around the center of the triangle, held in place by their mutual gravitation. What is the speed of each planet? [25 pts]

3 Bouncing balls [25 pts. total]



In the ACME rubber ball factory, a stream of rubber balls, each of mass m, comes out of a horizontal tube at a rate of R per second. These balls fall a distance of h into a bucket of mass M suspended by a rope from the ceiling. If the balls bounce out of the bucket back to their original height when leaving the tube, what is the (average) tension T in the massless rope holding the bucket? [25 pts.]

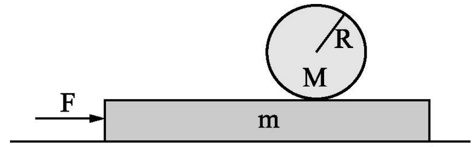
4 Simple Pendulum [25 pts. total]



Consider a simple pendulum consisting of a weight m swinging on a massless string of length L.
a) What is the frequency of the pendulum? You don't have to solve the full differential equation, but you have to show that the simple pendulum is a simple harmonic oscillator for small displacements from equilibrium. [15 pts]
b) If damping was added to the pendulum, qualitatively what are the three possible solutions to the differential equation? Please sketch the trajectories, and concisely describe the solutions (a sentence or two is sufficient). [10 pts]

5 Cylinder on Block [25 pts. total]

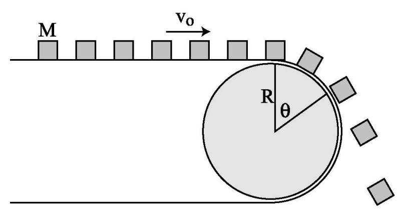
A uniform cylinder of mass M and radius R is at rest on a block of mass m. The block is at rest on a horizontal frictionless surface. If a horizontal force F is applied to the the block, it begins accelerating and the cylinder begins rolling without slipping (since there is friction between the block and the cylinder). The moment of inertia of a cylinder is $1/2\ MR^2$



Draw free-body diagrams of the block and cylinder, paying careful attention to where each force is applied. Calculate the acceleration of the block. [25 pts.]

6 Conveyor Belt [25 pts. total]

In a factory, boxes of mass M are carried along a conveyor belt moving at a constant speed v_0. The conveyor belt goes around a cylinder of radius R. There is a coefficient of static friction μ_s between the boxes and the conveyor belt.

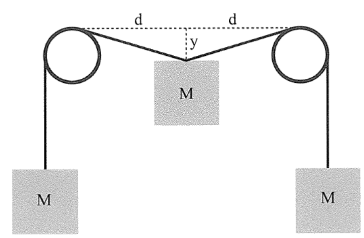


a) Set up the equation to determine at what angle θ_s the boxes begin to slip on the belt. You can leave this equation in terms of θ_s and the given constants– i.e. set up the equation, but you do not have to solve explicitly for θ_s. You can assume $\theta_s < \pi/2$. [13 pts]

b) Ignoring kinetic friction (assume $\mu_k = 0$), at what angle θ_c do the boxes leave contact with the conveyor belt? Leave your answer in terms of θ_s, i.e. do not try to plug in your detailed expression for θ_s from part (a). You can assume $\theta_c < \pi/2$. [12 pts]

7 Suspended blocks [25 pts. total]

Three blocks of equal mass M are suspended by two pulleys separated by a distance $2d$ as shown in the figure. The length of each of the two ropes is L. Ignore the mass of the ropes and the pulleys. The setup is left-right symmetric as shown in the figure.



a) Calculate the potential energy of the system as a function of the distance y. You are free to choose the zero or reference point of the potential energy. [9 pts.]

b) If the central mass M is moved to any arbitrary height y and let go, it will oscillate until friction brings it to rest. What is the value of y where the system comes to rest? Hint: sketch a potential energy diagram for this system. [9 pts.]

c) Draw a free-body diagram for the central mass when it comes to rest, and show that the total force on the central mass is zero. [7 pts.]