Name Key

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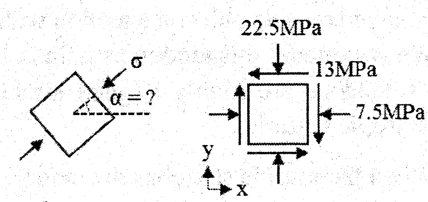
BioE 102 Midterm 1

1. **Stress Transformation** (35 points)

A single compressive stress, σ, is applied at an unknown angle. In the coordinate axes, this stress has normal and shear components of σ_{xx} = -7.5 MPa, σ_{xy} = -13 MPa, and σ_{yy} = -22.5 MPa.



a.) Determine the magnitude and angle of the applied stress. (15 points)

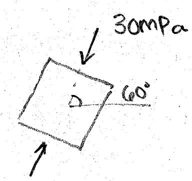
b.) If the material will fracture in compression at 43MPa and in shear at 18MPa, what is the minimum magnitude of σ required for fracture and due to which of these two failure modes? (20 points)

a.) If there is only a compressive stress it must be equal to σ_2 at an angle α_p

$$\alpha = \alpha_p = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}}\right) = \frac{1}{2}\tan^{-1}\left(\frac{-26}{-7.5+22.5}\right) = -30^\circ, +60^\circ$$

Since it's only a single compressive stress and it must result in the system above in x,y components, σ can be σ'xx(α=60) or σ'yy(α=-30)

$$\sigma'_{xx}(\alpha=60^\circ) = \sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = -30\text{MPa}$$



b.) $\sigma_{xx} = \sigma$

$\sigma_{1,2} = 0, \sigma$ ⇒ failure in compression requires $\sigma = -43$MPa

$$\tau_m = \pm\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = \pm\frac{\sigma_{xx}}{2} = \pm\frac{\sigma}{2}$$

⇒ failure in tension requires $\sigma = -36$MPa

⇒ will fail when $\sigma = -36$MPa due to shear stress

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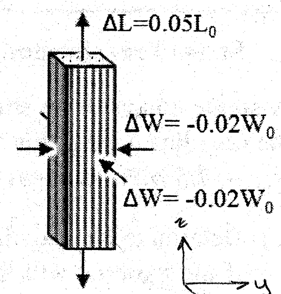
2. **Hook's Law** (35 points)



A flexed muscle pulls on a tendon with a tensile stress of σ_M causing it to extend. We will model this tendon as a linearly elastic homogeneous rectangular prism. As tendons are highly aligned structures, we will consider this a transverse isotropic material.

When the muscle stretches the tendon, it extends by 5% its original length along the longitudinal axis. Concurrently, the width of the tendon contracts by 2% the original width in both directions.

a.) Give expressions for all normal strains in terms of Poisson's ratios, moduli and tensile stress, σ_M. (10 points)
b.) Find expressions for a Poisson's ratio and Young's modulus for the tendon. (15 points)
c.) What is the maximum shear strain? Represent this shear strain in a free body diagram noting the magnitude of the shear strain and the angle of rotation for the free body diagram. (10 points)

a.) $\varepsilon_{xx} = \varepsilon_{yy} = \frac{-\nu'}{E'}\sigma_m$ $\qquad$ $\varepsilon_{zz} = \frac{1}{E'}\sigma_m$

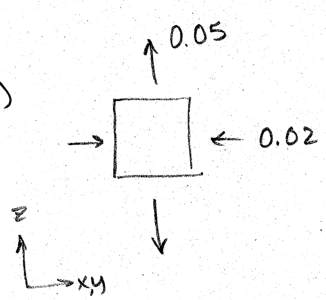
b.) $\varepsilon_{zz} = 0.05 = \frac{1}{E'}\sigma_m$

$E' = \frac{1}{0.05}\sigma_m = 20\sigma_m$

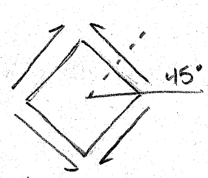
~~$\varepsilon_{xx} = -0.02 = \frac{-\nu'}{E'}\sigma_m$~~

$\nu' = \frac{0.02\,E'}{\sigma_m} = 0.4$

c.)



$\varepsilon_{xy,max}' = \pm\sqrt{\left(\frac{-0.02-0.05}{2}\right)^2} = \pm 0.035$ @ 45°



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3. **Strain Gauges** (30 points)

To test the accuracy of a set of strain gauges, you place them at 0°, 45° and 90° on an isotropic material with a Poisson's ratio of 0.2 and a Young's modulus of 3MPa. You stretch the material with a normal stress of 10kPa at an angle of $\alpha = 30°$ away from the 0° gauge.

a.) Solve for ε_0, ε_{45} and ε_{90} in terms of ε_{xx}, ε_{yy} and ε_{xy}. (12points)

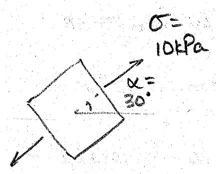
b.) Use Hook's law equations to calculate expected readings from each gauge. (12points)

c.) Given the linear approximation for strain necessary for our Hook's law equations, what may account (points) for deviation from our expected values? (*Assuming gauges are properly calibrated*)

a.) $\varepsilon_0 = \varepsilon_{xx}'(\alpha=0°) = \varepsilon_{xx}$

$\varepsilon_{90} = \varepsilon_{xx}'(\alpha = 90°) = \varepsilon_{yy}$

$\varepsilon_{45°} = \varepsilon_{xx}'(\alpha=45°) = \frac{\varepsilon_{xx}}{2} + \varepsilon_{xy} + \frac{\varepsilon_{yy}}{2}$



$\sigma_{xx} = \frac{\sigma}{2} + \frac{\sigma}{2}\cos(-2\alpha)$ =7.5

$\sigma_{yy} = \frac{\sigma}{2} - \frac{\sigma}{2}\cos(-2\alpha)$ =2.5

$\sigma_{xy} = \frac{-\sigma}{2}\sin(-2\alpha)$ =4.33

b.) $\varepsilon_{0°} = \varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right)$

$= \frac{1}{E}\left(\frac{\sigma}{2} + \frac{\sigma}{2}\cos(-2\alpha) - \nu\left(\frac{\sigma}{2} - \frac{\sigma}{2}\cos(-2\alpha)\right)\right)$

$= \frac{\sigma}{2E}\left(1-\nu + (1+\nu)\cos(-2\alpha)\right)$ $\qquad \varepsilon_{0°}\Big|_{\substack{\sigma=10kPa \\ \alpha=30°}} = 0.0023$

$\varepsilon_{90°} = \varepsilon_{yy} = \frac{1}{E}\left(\frac{\sigma}{2} - \frac{\sigma}{2}\cos(-2\alpha) - \nu\left(\frac{\sigma}{2} + \frac{\sigma}{2}\cos(-2\alpha)\right)\right)$

$= \frac{\sigma}{2E}\left(1-\nu - (1+\nu)\cos(-2\alpha)\right)$ $\qquad \varepsilon_{90°}\Big|_{\substack{\sigma=10kPa \\ \alpha=30°}} = 0.0003$

$\varepsilon_{45°} = \varepsilon_{xy} + \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}$

$\varepsilon_{xy} = \frac{1}{2G}\sigma_{xy} = \frac{-(1+\nu)\sigma}{2E}\sin(-2\alpha)$

$\Rightarrow \varepsilon_{45°} = \frac{-(1+\nu)\sigma}{2E}\sin(-2\alpha) + \underbrace{\frac{\sigma}{4E}\left(1-\nu+(1+\nu)\cos(-2\alpha)\right) + \frac{\sigma}{4E}\left(1-\nu-(1+\nu)\cos(-2\alpha)\right)}_{\text{from above}}$

$\varepsilon_{45°}\Big|_{\substack{\sigma=10kPa \\ \alpha=30°}} = 0.0031$

c.) Not accounting for higher order terms and assuming a LEHI material introduces possible sources of error in real-world measurements.