NE 180
Practice Midterm II
Fall Semester 2014
Solutions

November 4, 2014

1

The ITER device ($R = 6.2$ m, $a = 2.0$ m, $B_\phi = 5.3$ T, $q(0) = 1.0$) is assumed to operate with 40 MW of external heating and 80 MW of alpha-particle heating. Assume that the plasma has a surface area at $r = 0.9a$ of 612 m^2. Assume that at $r = 0.9a$ the safety factor q is 3.0 and that the ion temperature is 3.0 keV. Assume that the density is 8×10^{19} at this point and that the plasma is $Z = 1$ with a 50-50 D-T mixture. Assume that ten percent of the heat leaves though the ion channel, neglecting ohmic heating. Assume that both heating sources mentioned above deposit all of the heat inside the $r = 0.9a$ surface.

1. a.

At $r = 0.9a$, find τ_i, $\omega_{ci}\tau_i$, and $\kappa_\perp^i$ Also find the neoclassical collisionality parameter ν_i^*.

$$\boldsymbol{T_i(0.9\ a) = 3000\ \textbf{eV}}$$

$$\boldsymbol{n_i(0.9\ a) = 8.0 \times 10^{19}\ \textbf{m}^{-3} = 8.0 \times 10^{13}\ \textbf{cm}^{-3}}$$

$$n\tau_i = \frac{2.09 \cdot 10^7 (m_i/m_p)^{1/2} T_i^{3/2}}{\ln \Lambda} = 3.61 \times 10^{11} \text{ cm}^{-3}\text{s}$$

$$\boxed{\tau_i = 0.0045 \text{ s}}$$

$$\omega_{ci} = \frac{qB}{m_i} = 2.03 \times 10^8 \text{s}^{-1}$$

$$\boxed{\omega_{ci}\tau_i = 9.19 \times 10^5}$$

$$\boxed{\kappa_\perp^i = 2.0 \frac{nT_i\tau_i}{m_i(\omega_{ci}\tau_i)^2} = 9.80 \times 10^{16} \text{m}^{-1} \text{ s}^{-1}}$$

1. b.

For this value of ν_i^*, find the neoclassical collisionality regime; i. e. Pfirsch-Schlüter, Plateau. or Banana.

$$\nu^* = \frac{qR}{v_{th}\tau_i} = (3)(6.2)(T_i/m_i)^{-1/2}/\tau_i = \boxed{0.0121} \text{ so ions are}$$

$$\boxed{\text{Banana}}$$

1. c.

If the ion heat transport is neoclassical, find the value of ∇T_i at $r = 0.9a$.

$$q'' = (40 + 80) \times 10^6/612 = 19{,}607.8 \text{ Watt m}^{-2}$$

$$\text{Neoclassical Factor} = Q_{neo} = 2q^2\epsilon^{-3/2} = 109.545$$

$$\text{Since } q'' = -Q_{neo}\kappa_\perp \frac{dT(r)}{dr}, \text{ we have}$$

$$\frac{dT(r)}{dr} = 1.8 \times 10^{-15} \text{Jm}^{-1} = \boxed{11.35 \text{ keV m}^{-1}}$$

1. d.

At the center of the device, the electron temperature is 25.0 keV and the ions are at 15.0 keV. The density there is 1.1×10^{20} m^{-3}. Find the electron-ion interspecies heating at $r = 0$, in megawatts per cubic meter.

$$\boldsymbol{Q_{ie} = \frac{3m_e}{m_i}\frac{n(T_i - T_e)}{\tau_e}}$$

$\tau_e = 2.0 \times 10^{10} T_e^{3/2}/n_e$ with T_e in eV and MKS density. Then $\tau_e = 718\mu$s. and

$$\boldsymbol{Q_{ie} = \boxed{160 \text{ kW m}^{-3}}}$$

1. e.

Find the volumetric ohmic heating at $r = 0$. Give the answer in megawatts per cubic meter. Hint: note that the toroidal current can be derived from q:

$$J_\phi(0) = \frac{2\,B_\phi(0)}{\mu_0 q(0) R_0}$$

.

$$\boldsymbol{\eta = 2.8 \cdot 10^{-8} Z T_e^{-3/2}}$$

$$\boldsymbol{= 2.24 \times 10^{-10}\ \Omega\,\text{m}}$$

$$\boldsymbol{J_\phi(0) = \frac{2\,B_\phi(0)}{\mu_0 q(0) R_0} = 1.36 \times 10^{6} \text{ A m}^{-2}}$$

$$\boldsymbol{\boxed{P_\Omega = 414.6 \text{ W m}^{-3}}}$$

2

A hohlraum design for NIF is made with depleted uranium ($M_U = 238m_p$). The interior of the hohlraum is illuminated with frequency-tripled neodymium-glass laser light, $\lambda = 0.35\ \mu$. There are two laser entrance holes (LEHs), each with a 2.0 mm diameter. The total laser power is 4.0×10^{14} W with a duration of 4×10^{-9} s.

2. a.

If the fraction of the power loss due to blackbody radiation through the LEHs is thirty percent, find the photon temperature in the hohlraum.

Radiation loss:

$$\boldsymbol{q'' = 0.3P_{laser}/A_{hole} = \frac{0.3 \times 4 \times 10^{14}}{\left(2 \cdot \left(\frac{\pi d^2}{4}\right)\right)} = 1.91 \times 10^{15}\ \mathrm{W\ cm^{-2}}}$$

$\boldsymbol{q''_{BB} = \sigma T_\gamma^4 = 1.03 \times 10^5 T^4 = 1.9 \times 10^{15}\ \mathrm{W\ cm^{-2}}}$. **Then** $\boldsymbol{T_\gamma = \left(\frac{1.9 \times 10^{15}}{1.03 \times 10^5}\right)^{1/4}}$**, or**

$$\boxed{\boldsymbol{T_\gamma = 369\ \mathrm{eV}}}$$

2. b.

Assume that the electron temperature at the hohlraum wall is 3.0 keV. Assume that the average charge $< Z >$ of the uranium is 30 and that the adiabatic index γ_e for the electrons is 1.0. Find the ion-acoustic wave speed c_s in the uranium plasma. Ignore the uranium ion temperature.

The ion-acoustic wave speed is given by:

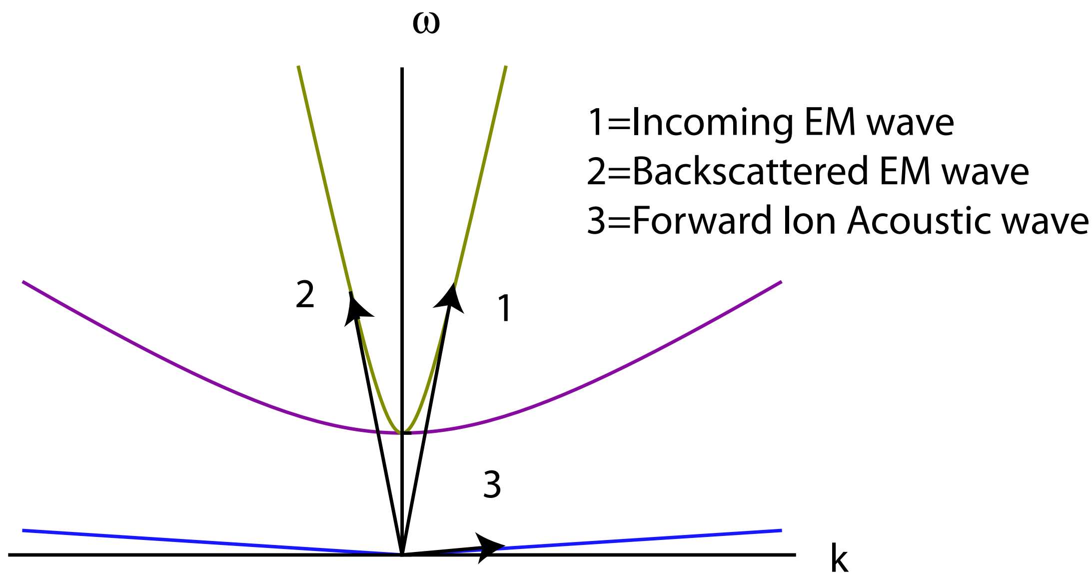
$$\boldsymbol{c_s = \left(\frac{\gamma_e Z T_e + \gamma_i T_i}{m_i}\right)^{1/2}}$$

Using $Z = 30$ and $m = 238m_p = 2.38 \times 1.67 \times 10^{-27}$ with $T_e = 3000 \times 1.6 \times 10^{-19}$ gives

$$\boxed{c_s = 1.90 \times 10^5 \ \ \mathrm{m\ s^{-1}}}$$

2. c.

Draw a Stokes diagram for stimulated Brillouin scattering (SBS), labeling the forward (1) and backscattered (2) electromagnetic wave and the ion acoustic wave (3).



2. d.

For SBS taking place where the plasma density is at 0.8× the critical density, find the value of the electron density there.

The critical density n_c is defined as $\omega_{pe}(n_c) = \omega_L$, or

$$n_c = \frac{m_e \epsilon_0 \omega_L^2}{e^2}$$

Here $\boldsymbol{\omega_L = \frac{2\pi c}{\lambda_L} = \frac{2\pi \cdot 3 \times 10^8}{0.35 \times 10^{-6}} = 5.39 \times 10^{15}\ \ \mathrm{s}^{-1}}$**. Then** $\boldsymbol{n_c = 9.13 \times 10^{27}\ \mathrm{m}^{-3}}$ **and thus**

$$\boxed{\boldsymbol{n = 0.8 n_c = 7.31 \times 10^{27}\ \mathrm{m}^{-3}}}$$

2. e.

Find the wavelength shift $\Delta\lambda$ for the backscattered electromagnetic wave by the following method: to first order, the wavenumber $k_3 = 2k_1$. Then find $\omega_3 = k_3 c_s$ and use this to obtain $\omega_2 = \omega_1 - \omega_3$. Then solve for the (free-space) wavelength shift using $\Delta\lambda/\lambda \approx -\Delta\omega/\omega = -\omega_3/\omega_1$. Express your answer in angstroms. ($1\mathring{A} = 10^{-10}$ m.)

For the incoming EM wave

$$\boldsymbol{\omega_L^2 = \omega_{pe}^2 + k_1^2 c^2}$$

Noting that $\boldsymbol{\omega_{pe}^2/\omega^2 = n/n_c}$ **and** $\boldsymbol{k_0 = \omega/c}$**, this can be re-written as**

$$\boldsymbol{k_1 = k_0\sqrt{1 - n/n_c}}$$

and thus $\boldsymbol{k = \sqrt{1-0.8} \cdot \frac{2\pi c}{\lambda_0} = 8.03 \times 10^6\ \mathrm{m}^{-1}}$**. Then** $\boldsymbol{\omega_3 \approx 2k_1 c_s = 3.056 \times 10^{12}\ \ \mathrm{s}^{-1}}$**. Then** $\boldsymbol{\Delta\omega/\omega_L = -5.675 \times 10^{-4}}$ **and**

$$\boxed{\boldsymbol{\Delta\lambda = 3500 \cdot 5.675 \times 10^{-4}\ \ \mathring{\mathrm{A}} = 1.98\ \mathring{\mathrm{A}}}}$$