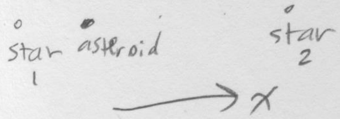
DeWeese spring 2014 7A lecture Section 3 final exam

1.a.) $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = -\frac{GM_AM_s}{D^2}\hat{i} + \frac{GM_AM_s}{(3D)^2}\hat{i} = -\frac{GM_AM_s}{D^2}\left(1-\frac{1}{9}\right)\hat{i}$



$= \boxed{-\frac{8}{9}\frac{GM_AM_s}{D^2}\hat{i}}$

(force points to left in diagram)

b.) cons. of E.:

$\left(PE_g + \cancelto{0}{\tfrac{1}{2}mv^2}\right)_f - \left(PE_g + \cancelto{0}{\tfrac{1}{2}mv^2}\right)_i = W_{ext}$

$\left(-\frac{GM_AM_s}{2D} - \frac{GM_AM_s}{2D}\right) - \left(-\frac{GM_AM_s}{D} - \frac{GM_AM_s}{3D}\right) = W_{ext}$

work by rocket $= W_{ext} = \frac{GM_AM_s}{D}\left(-\frac{1}{2} - \frac{1}{2} + 1 + \frac{1}{3}\right) = \boxed{\frac{1}{3}\frac{GM_AM_s}{D}}$

$W_{ext} > 0$ ✓

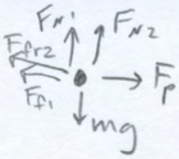
c.) $W = F_x \Delta x$ for constant force

$W_{total} = \frac{1}{3}\frac{GM_AM_s}{D} = F_{ix}\frac{D}{2} + F_{\substack{2nd\\half\\x}}\frac{D}{2}$

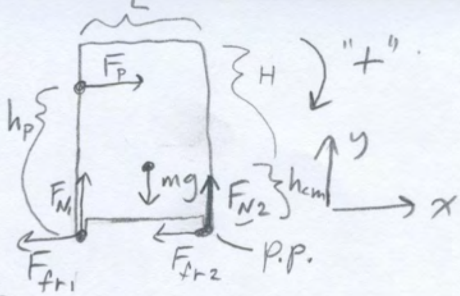
$\boxed{F_{\substack{2nd\\half\\x}} = -F_{ix} + \frac{2}{3}\frac{GM_AM_s}{D^2}}$

d.) $\vec{J}_{net} = \Delta\vec{P} = \boxed{0}$

2a.) FBD dresser



FBD$_{extended}$: dresser



b.) N2L y: $-mg + F_{N1} + F_{N2} = m\cancel{a_y}^{0}$ ← 2 unknowns... (1)

N2L$_\alpha$: $\tau_p + \cancel{\tau_{fr1}}^{0} + \cancel{\tau_{fr2}}^{0} + \tau_{N1} + \cancel{\tau_{N2}}^{0} + \tau_g = \cancel{I\alpha}^{0}$

$$F_p h_p - \frac{mgL}{2} + F_{N1} L = 0$$

$$\boxed{F_{N1} = \frac{mg}{2} - \frac{F_p h_p}{L}} \quad (2)$$

c.) (2) in (1) ⇒ $F_{N2} = mg - F_{N1} = mg - \frac{mg}{2} + \frac{F_p h_p}{L}$

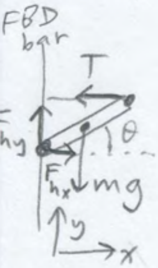
$$= \boxed{\frac{mg}{2} + \frac{F_p h_p}{L}}$$

d.) F_{max} is achieved when $F_{N1} = 0$ & $F_{N2} = mg$:

(2) ⇒ $0 = \frac{mg}{2} - \frac{F_{max} h_p}{L}$ ⇒ $\boxed{F_{max} = \frac{mgL}{2h_p}}$

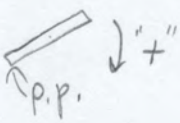
3. a.) N2L y: $F_{hy} - mg = m\cancel{a_y}^{\,0} \Rightarrow \boxed{F_{hy} = mg}$ "+" ⇒ upward

FBD bar

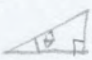


b.) N2L x: $-T + F_{hx} = m\cancel{a_x}^{\,0}$ ← 2 unknowns...

$N2L_\alpha$: $mg\frac{L}{2}\cos(\theta) - T\cancel{L}\sin\theta = I\cancel{\alpha}^{\,0}$



$$T = \frac{mg}{2}\frac{\cos(\theta)}{\sin(\theta)} = \boxed{\frac{mg}{2}\cot(\theta)}$$



c.) $N2L_\alpha$: $mg\frac{L}{2}\cos(\theta) = I_{pp}\alpha$

$$= \frac{mL^2}{3}\alpha$$

$$I_{pp} = \int_0^L \underbrace{dr\,\rho}_{dm}\, r^2 = \frac{\rho L^3}{3} = \frac{mL^2}{3}$$

⇑ (since $\rho = \frac{M}{L}$)

$$\alpha = \frac{\cancel{mg}\cancel{L}}{2}\cos(\theta)\frac{3}{\cancel{m}L^{\cancel{2}}}$$

$$= \boxed{\frac{3g}{2L}\cos(\theta)}$$ "+" ⇒ clockwise

d.) C.o.E.: $(PE_g + KE + KE_\alpha)_f - (\cancel{PE_g}^{\,0} + \cancel{KE}^{\,0} + \cancel{KE_\alpha}^{\,0})_i = \cancel{W_{ext}}^{\,0}$

set $PE_g = 0$ for initial location of C. of Mass

0 because hinge acts on stationary part of bar...

$$mg\underbrace{\left(-\frac{L}{2} - \frac{L}{2}\sin(\theta)\right)}_{\text{change in height of c.m.}} + \frac{1}{2}M v_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = 0 \quad (1)$$

parallel axis theorem:

$$I_{cm} = I_{pp} - M\left(\frac{L}{2}\right)^2$$

$$= \left(\frac{1}{3} - \frac{1}{4}\right)ML^2$$

$$= \frac{1}{12}ML^2 \quad (2)$$

$$|\vec{v}_{cm}| = \frac{L}{2}\omega \quad (3)$$

(2) & (3) in (1) ⇒ $-\frac{\cancel{m}gL}{2}(1+\sin(\theta)) + \frac{1}{2}\cancel{M}v_{cm}^2 + \frac{1}{2}\frac{1}{12}\cancel{ML^2}v_{cm}^2\frac{4}{\cancel{L^2}} = 0$

$$-\frac{gL}{\cancel{2}}(1+\sin(\theta)) + \frac{1}{\cancel{2}}v_{cm}^2 + \frac{1}{\cancel{2}}\frac{1}{3}v_{cm}^2 = 0$$

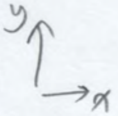
$$v_{cm}^2\left(1+\frac{1}{3}\right) = gL(1+\sin(\theta))$$

$$\boxed{v_{cm} = \sqrt{\frac{3}{4}gL(1+\sin(\theta))}}$$

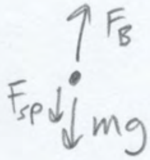
e.) at its lowest point, $\alpha = 0$ since there is no net torque on bar so accelleration is only radial, like uniform circular motion

$$\vec{a}_{cm} = a_y^{cm}\hat{j} = \frac{v_{cm}^2}{R}\hat{j} = \frac{2}{L}\frac{3}{4}gL(1+\sin(\theta))\hat{j} = \boxed{\frac{3}{2}g(1+\sin(\theta))\hat{j}}$$ upward

4. a.) $F_B = \rho_{water}\, g\, V_{displaced} = \boxed{\rho_{water}\, g\, \frac{h}{2}\pi r^2}$ upwards $\qquad \rho_w \equiv \rho_{water}$



b.) FBD$_{buoy}$



N2Ly: $F_B - F_{sp} - mg = m\cancel{a_y}^{\,0}$

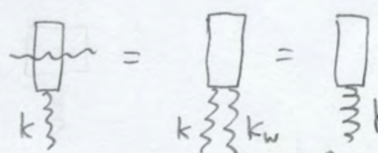
$$\rho_w g \frac{h}{2}\pi r^2 + k\Delta y - \frac{1}{4}\rho_w h \pi r^2 g = 0$$

$$\Delta y = \frac{1}{k}\left(\frac{1}{4} - \frac{1}{2}\right)\rho_w g h \pi r^2$$

$$= -\frac{\rho_w g h \pi r^2}{4k}$$

so unstretched spring length $= L_0 = \boxed{D - \frac{\rho_w g h \pi r^2}{4k}}$

c.) S.H.O. : $\omega = \sqrt{\frac{k_{eff.}}{m}}$ — 1



$k_{eff} = k + k_w$

N2Ly$_{buoy}$: $\underbrace{F_B - F_{sp}}_{\text{forces add}} - mg = ma_y$

Hooke's Law: $k_w = \frac{|F_B|}{|\Delta y|} = \frac{\rho_w g \frac{h}{2}\pi r^2}{\frac{h}{2} - \frac{h}{4}} = \frac{4}{h}\rho_w g \frac{h}{2}\pi r^2 = 2\rho_w g \pi r^2$

← since $\Delta y = 0$ occurs when buoy is $\frac{1}{4}$ submerged

$k_{eff.} = k + k_w = k + 2\rho_w g \pi r^2$

one full oscillation takes a time of $T = \frac{1}{f} = \frac{2\pi}{\omega} \overset{(1)}{=} 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{\frac{1}{4}\rho_w \pi r^2 h}{k + 2\rho_w g \pi r^2}}$

$$\boxed{T = \pi\sqrt{\frac{\rho_w \pi r^2 h}{k + 2\rho_w g \pi r^2}}}$$

(actually this is flawed because the buoy would leave the water before completing 1 cycle ...)

d.) $F_{bottom} = P_{bottom} A_{tank}$ $\qquad A_{tank} = \pi R^2$

BE: $P_{bottom} = 1\,atm + \rho_w g\left(H + \frac{h}{2} + \Delta h\right)$ where $\Delta h(\pi R^2 - \pi r^2) = \frac{h}{2}\pi r^2$

$$\Delta h = \frac{\frac{h}{2} r^2}{R^2 - r^2} = \frac{h r^2}{2(R^2 - r^2)}$$

$$\boxed{F_{bottom} = \left[1\,atm + \rho_w g\left(H + \frac{h}{2} + \frac{h r^2}{2(R^2 - r^2)}\right)\right]\pi R^2}$$

$r \ll R \Rightarrow F_{bottom} = 1\,atm \cdot \pi R^2 + \pi \rho_w g\left(HR^2 + \frac{h}{2}(R^2 + r^2)\right)$

5. a.) $v_{string} = \sqrt{F_T/\mu} \Rightarrow \boxed{F_T = v_{string}^2 \, \mu}$

wavespeed

$\lambda_{string} = 2L \Rightarrow f = \dfrac{v_{string}}{\lambda_{string}}$

$f = \dfrac{v_{string}}{2L}$

b.) $\lambda_{air} = \dfrac{v_{snd}}{f} = \boxed{\dfrac{v_{snd}}{v_{string}} 2L}$

c.) Doppler shift } moving observer

approaching

$f' = f\left(1 + \dfrac{v_{obs}}{v_{snd}}\right)$

$= \boxed{\dfrac{v_{string}}{2L}\left(1 + \dfrac{v_{skate}}{v_{snd}}\right)}$

d.) Intensity = $\dfrac{\text{Power}}{\text{Area}} \Rightarrow P = I \cdot A = \boxed{I \, 4\pi D^2}$

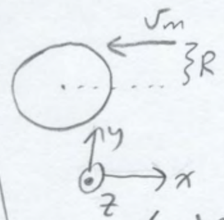
6. b.) cons. of angular momentum:

$$\vec{L}_{i_{tot.}} = \cancel{\vec{L}_{i_{probe}}}^{\,0} + \vec{L}_{i_{meteoroid}} = R v_m M_m \hat{k}$$

$$\vec{L}_{f_{tot}} = \underset{\Uparrow}{I}_{cm}\vec{\omega} = I_{cm}\,\omega\,\hat{k}$$

spherical symmetry & axis of rot. through c.m.

$$\Rightarrow \boxed{\omega = \frac{R v_m M_m}{I_{cm}}}$$

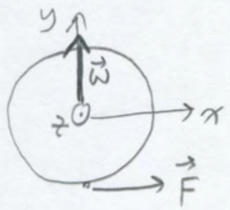


(note that I will use different axes for part c.)

a.) cons. of linear momentum $\hat{x}$: $-M_m v_{m_x} + 0 = (M_m + M_p)\, v_{cm_{p\&m\,x}}$

$$v_{cm_{p\&m\,x}} = \frac{-M_m v_m}{M_m + M_p} \qquad \text{so speed is} \qquad \boxed{v_{cm_{p\&m}} = \frac{M_m v_m}{M_m + M_p}}$$

c.) new coords:



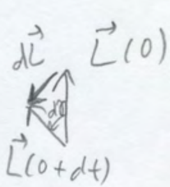
$$N2L_\alpha \quad \vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$$\vec{\tau}_{net} = \vec{\tau}_{rocket} = \vec{r} \times \vec{F}_{rocket} = (0,-R,0)\times(F,0,0)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -R & 0 \\ F & 0 & 0 \end{vmatrix} = -(F)(-R)\hat{k} = FR\hat{k} \text{ (out of the paper)}$$

$$\text{so } \boxed{\frac{d\vec{L}}{dt} = FR\hat{k}}$$

d.)



$$\Omega \equiv (\text{rate of procession of } \vec{L}) = \frac{d\gamma}{dt} \underset{\Uparrow}{=} \frac{dL/dt}{L} = \frac{FR}{L}$$

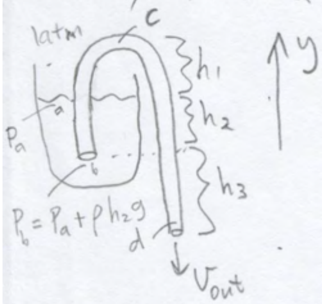
$$d\gamma = \frac{dL}{L}$$

$$t = x/v$$

Δt = time to process by $90° = \frac{1}{4}T$

$$\Delta t = \frac{\pi/2}{FR/L} = \boxed{\frac{\pi L}{2FR}}$$

7. a.) B.E. $P_a + \frac{1}{2}\rho \cancel{v_a^2}^{\to 0} + \rho g y_a = P_d + \frac{1}{2}\rho v_d^2 + \rho g y_d$ ①



$v_a \ll v_d$ $\quad$ $P_a = 1atm$ (absolute Pressure)

N2L $\Rightarrow P_d = 1atm$

so ① $\Rightarrow$ $\frac{1}{2}\rho v_d^2 = \rho g(y_a - y_d) = \rho g(h_2 + h_3)$

$$v_d^2 = 2g(h_2+h_3)$$

$$\boxed{v_d = \sqrt{2g(h_2+h_3)}}$$

b.) continuity Eq: $A_c v_c \rho_c = A_d v_d \rho_d$

incompressible $\Rightarrow \rho_c = \rho_d \Rightarrow A_c v_c = A_d v_d$

$$\boxed{v_c = v_d = \sqrt{2g(h_2+h_3)}}$$

c.) B.E. $P_a + \frac{1}{2}\rho \cancel{v_a^2}^{\to 0} + \rho g y_a = P_c + \frac{1}{2}\rho v_c^2 + \rho g y_c$

$1atm + 0 + \rho g(y_a - y_c) = P_c + \frac{1}{2}\rho v_c^2$

$P_c = 1atm - \rho g h_1 - \frac{1}{\cancel{2}}\rho \cancel{2}g(h_2+h_3)$

$= \boxed{1atm - \rho g(h_1+h_2+h_3)}$ absolute pressure

d.) P_c must be ≥ 0 in absolute pressure, so

$1atm - \rho g(h_{1max} + h_2 + h_3) = 0$

$$h_{1max} = \boxed{\frac{1atm}{\rho g} - h_2 - h_3}$$