Physics 7B, Fall 2007, Section 2, Instructor: Prof. Adrian Lee
First Midterm Examination, Tuesday October 2, 2007

Please do work in your bluebooks. You may use one double-sided 3.5" x 5" index card of notes. Test duration is 110 minutes.

(Giancoli Ch 20, problem 45)

1) Two samples of an ideal gas are initially at the same temperature and pressure; they are each compressed reversibly from a volume V to a volume V/2, one isothermally, the other adiabatically. (30 points total)

a) In which sample is the final pressure greater? (10 pts)

b) Determine the change in entropy of the gas for each process. (10 pts)

c) What is the entropy change of the environment for each process? (10 pts)

2) Three Independent Questions

a) **Heat Conduction.** A metal bar has a length L and has a uniform cross section. One end of the bar is held at 100˚C and the other is placed in an ice-water mix. It takes 12 minutes for the bar to conduct enough heat to melt one kilogram of ice. How long would it take a uniform bar of the same metal and the same volume but of length 2L to melt one kilogram of ice? (10 pts)

b) **Radiative transfer.** On a hot day, the solar radiation incident upon a black surface is 1100 W/m^2. If the surface acts as a perfectly radiating body, what temperature does it come to? $\sigma = 5.67 \times 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$ (10 pts)

c) **Kinetic theory of gases.** Is the total translational kinetic energy of all the molecules in a volume V of air at atmospheric pressure larger, smaller, or the same on a hot day as on a cold day? Give a brief explanation using simple equations. (10 pts)

3) The Stirling Cycle consists of (i) an isothermal expansion at $T = T_H$, (ii) a constant volume reduction in pressure at $V = V_a$, (iii) an isothermal compression at $T = T_L$, and finally (iv) a constant volume increase in pressure at $V = V_b$ to the starting point. In this problem, you will calculate the efficiency of this type of engine. Assume you know T_L, T_H, the two volumes (V_a and V_b), and that the specific heat is given by $C_V = 3/2R$ for the monatomic gas used in the engine.

a) Sketch the process in a P-V diagram. What is the heat and work during the two isothermal stages? (10 pts)

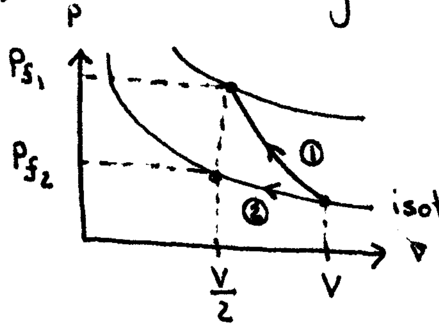
b) What is the heat and work during the two constant volume stages? (10 pts)

c) What is the net work, net heat, and the efficiency of the entire process? (10 pts) (for partial credit, express the efficiency in terms of net work and net heat without solving for the two expressions) (10 pts)

① $\{1\}$ $\{2\}$ $\left.\begin{array}{l} T_{10} = T_{20} = T_0 \\ P_{10} = P_{20} = P_0 \end{array}\right\}$ initially both gases @ same T & P.

Reversible compression $V \longrightarrow \frac{V}{2}$ $\begin{array}{l} 1 \longrightarrow \text{adiabatically} \\ 2 \longrightarrow \text{isothermally} \end{array}$

a) Which is greater P_{f_1} or P_{f_2} ?



We can see from the PV diagram that we should find $P_{f_1} > P_{f_2}$

1. $P_0 V_0^{\gamma} = P_{f_1} V_f^{\gamma}$

$P_0 V_0^{\gamma} = P_{f_1} \left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P_0 = P_{f_1} \frac{1}{2^{\gamma}} \Rightarrow P_{f_1} = 2^{\gamma} P_0$

$\gamma = \frac{d+2}{d} > 1$

2. $P_0 V_0 = P_{f_2} V_f = P_{f_2} \left(\frac{V_0}{2}\right) \Rightarrow P_{f_2} = 2 P_0$

$\boxed{P_{f_1} > P_{f_2}}$ P_f greater for adiabatic

b) Change in S of gas for each process

1. $\Delta Q = 0$ $\Delta S = \int \frac{dQ}{T} = 0 \Rightarrow \boxed{\Delta S_1 = 0}$

2. $\Delta S_2 = \int \frac{dQ}{T} = \frac{1}{T} \Delta Q$

$\Delta U = \Delta Q - \Delta W = 0 \Rightarrow \Delta Q = \Delta W$

$\Delta U = \frac{d}{2} NK \Delta T = 0$

$\Delta W = \int P dV = NKT \int_V^{V/2} \frac{dV}{V}$

$= NKT \ln \left|\frac{V/2}{V}\right| = NKT \ln|\tfrac{1}{2}|$

$= -NKT \ln 2$

$= -nRT \ln 2$

$\Delta S_2 = \frac{1}{T} \Delta Q = \frac{1}{T} \Delta W = \frac{1}{T}(-nRT\ln 2)$

$\boxed{\Delta S_2 = -nR\ln 2}$

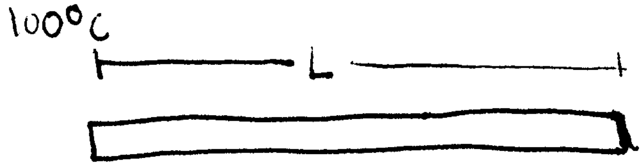
c) $\Delta S_{Tot} = \cancel{\Delta S} \; \Delta S_E + \Delta S_{sys} = 0$ ($\Delta S_{Tot} = 0$ for reversible process)

$\Rightarrow \Delta S_E = -\Delta S_{sys}$ $\Delta S_E = \Delta S$ Environment

$\boxed{\begin{array}{l} \Delta S_{E_1} = 0 \\ \Delta S_{E_2} = nR\ln 2 \end{array}}$

2) a) Heat Conduction

0°C (Temp of Ice Water)



$t = 12$ minutes to melt 1 kg of Ice

12 minutes = 720 seconds

How long to melt 1kg of Ice if the bar was made of the same metal, but had length 2ℓ, and the volume was the same?

This means we stretch the bar from L to 2L

$$V_0 = A_0 L = A_1 2L = V_1 \quad \therefore \quad A_0 \cancel{L} = A_1 2 \cancel{L}$$

$$\boxed{A_1 = \frac{A_0}{2}}$$

In the first case:

$$H = \frac{\Delta Q}{\Delta t} = \frac{mL}{\Delta t} = \frac{1\cancel{kg}(3.3 \times 10^5\ J/\cancel{kg})}{720\ s} = 458\ J/s$$

$$H = \frac{KA}{L}(T_2 - T_1) = 458\ J/s$$

$$\therefore \quad K = \frac{L}{A}\underbrace{\frac{1}{(T_2 - T_1)}}_{=100} 458\ J/s$$

$$\boxed{K = 4.58\ \frac{L}{A}\frac{J}{s}}$$

Lee MT1 Problem 2 Solution

In the second case $K_1 = K_1$

$$H_1 = \frac{K A_1}{L_1}(T_2 - T_1)$$

$$= 4.58 \frac{\cancel{L}}{\cancel{A}} \frac{\cancel{A}/2}{2\cancel{L}}(T_2 - T_1)$$

or $H_1 = \frac{K A_1}{L_1}(T_2 - T_1)$

$$= K \frac{A/2}{2L}(T_2 - T_1)$$

$$= \frac{1}{4} K \frac{A}{L}(T_2 - T_1)$$

$$\boxed{H_1 = \frac{4.58}{4}\underbrace{(T_2 - T_1)}_{=100} = 114.5\ J/s} = \boxed{\frac{1}{4} H_0 = H_1}$$

$$H_1 = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$$

$$\Delta t = \frac{\Delta Q}{H} = \frac{mL}{H} = \frac{(1\,kg)(3.3 \times 10^5\ J/kg)}{114.5\ J/s}$$

$$\boxed{\Delta t = 2.880 \times 10^3\ s}$$

or since H_1 is 4 times slower the 1 kg of ice will ~~take~~ melt in 4 times ~~long~~ the time

$$\boxed{\Delta t = 4(12) = 48 \text{ minutes} = 2.880 \times 10^3\ s}$$

b) Radiative Transfer

$S = \frac{W}{m^2}$

$= 1100\ W/m^2$

Pefect Black body $\varepsilon = 1$

$\sigma = 5.67 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$

The object will have a constant temperature when the $P_{absorbed} = P_{radiated}$

$P_{absorbed} = \varepsilon A_{cs} S$ $\qquad$ $P_{rad} = \sigma \varepsilon A_{sur} T^4$

Set equal

$$\cancel{\varepsilon} A_{cs} S = \sigma \cancel{\varepsilon} A_{sur} T^4$$

$$T^4 = \frac{A_{cs}}{A_{sur}} \frac{S}{\sigma} \implies T = \sqrt[4]{\frac{A_{cs}}{A_{sur}}} \sqrt[4]{S/\sigma}$$

However since its a <u>surface</u> it is 2D so

$A_{cs} = A_{sur}$

$$\therefore \quad T = \sqrt[4]{S/\sigma} = \sqrt[4]{\frac{1100\ \cancel{W}/\cancel{m^2}}{5.67 \times 10^{-8}\ \cancel{W}\,\cancel{m^{-2}}\,K^{-4}}} = \sqrt[4]{1.94 \times 10^{10}\ K^4}$$

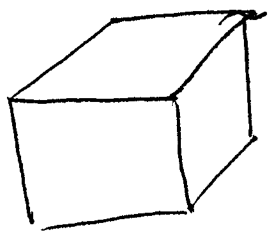
$$= \sqrt[4]{1.94 \times 10^{+10}\ K^4} = 373^{\circ}\ K$$

$$\boxed{T = 373^{\circ}\ K}$$

c) Kinetic Theory

Is the <u>total</u> translational kinetic energy of all the molecules in a volume V larger, smaller, or the same on a hot day as on a cold day? Explain.

Consider drawing a box at 1 atm on a cold day T_c



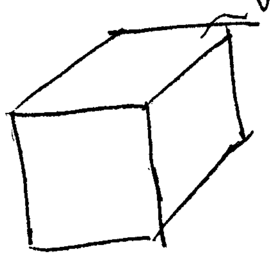
Volume V

$$PV = N_c k T_c \Rightarrow N_c = \frac{PV}{kT_c}$$

$$T_c = \frac{PV}{N_c k}$$

and on the hot day T_H

Volume V



$$PV = N_H k T_H \Rightarrow N_H = \frac{PV}{kT_H}$$

$$T_H = \frac{PV}{N_H k}$$

since P, V are equal in both cases, but $T_H > T_c$ so

$N_c > N_H$

By the equipartition theorem the total translational energy of the gas is

$$U_{TOT} = \frac{3}{2} N K T$$

Hot Day

$$U_{TOT} = \frac{3}{2} N_H k T_H$$

$$= \frac{3}{2} \frac{PV}{\cancel{k T_H}} \cancel{k T_H}$$

$$= \frac{3}{2} PV$$

Cold Day

$$U_{TOT} = \frac{3}{2} N_c k T_c$$

$$= \frac{3}{2} \frac{PV}{\cancel{k T_c}} \cancel{k T_c}$$

$$U_{TOT} = \frac{3}{2} PV$$

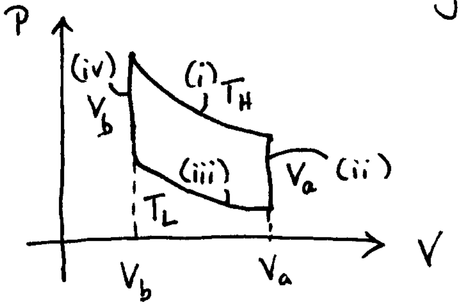
They both equal $\frac{3}{2} PV$. They are the same!

③ (i) isothermal expansion at T_H
(ii) isochoric depressurization at $V = V_a$
(iii) isothermal compression at T_L
(iv) isochoric increase in pressure at $V = V_b$.

$C_v = \frac{3}{2}R$.

We will find the efficiency.

(a) Draw a PV diagram.



What is the heat and work during the isothermal stages?

(i) $W_{(i)} = \int P dV \underset{\substack{\uparrow \\ \text{ideal gas law}}}{=} \int_{V_b}^{V_a} NkT_H \frac{dV}{V} = \boxed{NkT_H \ln\left(\frac{V_a}{V_b}\right)} = nRT_H \ln\left(\frac{V_a}{V_b}\right)$

0 for isothermal processes

First law $\Rightarrow$ $Q_{(i)} = \cancel{\Delta E} + W = \boxed{NkT_H \ln\left(\frac{V_a}{V_b}\right)}$.

(iii) $W_{(iii)} = \int P dV = NkT_L \ln\left(\frac{V_b}{V_a}\right) = \boxed{-NkT_L \ln\left(\frac{V_a}{V_b}\right)}$

$Q_{(iii)} = W_{(iii)} = \boxed{-NkT_L \ln\left(\frac{V_a}{V_b}\right)} = -nRT_L \ln\left(\frac{V_a}{V_b}\right)$

~~(iii)~~

(b) For isochoric processes $dV = 0 \Rightarrow \boxed{W = 0}$.

In the first law, $\Delta E = Q - \cancelto{0}{W} = Q$.

Now, $E = nC_vT$, so $\Delta E = nC_v\Delta T$

$Q_{(ii)} = \Delta E_{(ii)} = \boxed{nC_v(T_L - T_H)} = \boxed{-nC_v(T_H - T_L)}$

$Q_{(iv)} = \Delta E_{(iv)} = nC_v(T_H - T_L) = \boxed{+nC_v(T_H - T_L)}$

(c) $W_{net} = W_{(i)} + W_{(iii)} = nR(T_H - T_L)\ln\left(\frac{V_a}{V_b}\right)$

$Q_{in} = Q_{(i)} + Q_{(iv)} = nRT_H \ln\left(\frac{V_a}{V_b}\right) + nC_V(T_H - T_L)$

Efficiency is given by

$$\boxed{e = \frac{W_{net}}{Q_{in}} = \frac{nR(T_H - T_L)\ln\left(\frac{V_a}{V_b}\right)}{nRT_H\ln\left(\frac{V_a}{V_b}\right) + nC_V(T_H - T_L)}}$$