1 Problem 1

Volume Expansion: $\Delta V = \beta \Delta T V_0$

1.1 Part A

For Aluminum: $\alpha_{al} = 25 \times 10^{-6}/\mathrm{K}$

In case of small change in volume: $\beta \approx 3\alpha$
$\beta_{al} = 75 \times 10^{-6}/\mathrm{K}$
$V_f = \beta(T - T_0)V_0 + V_0$
$V_f = V_0(75 \times 10^{-6}(T - T_0) + 1)$

1.2 Part B

For H2O: $\beta_w = 210 \times 10^{-6}/\mathrm{K}$

$\Delta V_w = 210 \times 10^{-6} \Delta T V_0$
$V_f = V_0(210 \times 10^{-6}(T - T_0) + 1)$

1.3 Part C

The volume expansion coefficient of water is larger than that of the aluminum container. In essence, for a change in temperature, the water will expand more than the container. Because the water is already at the top of the container, it spills out with a raise in temperature.

1.4 Part D

To avoid spillage the volume expansion coefficient of the container must be equal (or greater) than that of water. $\beta_w \leq \beta_c$ For small changes in volume:

$\beta_w \leq 3\alpha_c$ $210 \times 10^{-6}/\,\mathrm{K} \leq 3\alpha_c$
$70 \times 10^{-6}/\,\mathrm{K} \ \leq \alpha_c$

The linear coefficient of expansion of the container must be greater than $70 \times 10^{-6}/\,\mathrm{K}$

2 Problem 2

2.1 Part A

Is it possible for a liquid to vaporize below the boiling point? Explain.

Yes, it is possible for liquids significantly below boiling point to vaporize. This is the familiar phenomena of evaporation. Even in temperatures below boiling point, a fraction of the molecules composing the liquid will have sufficient kinetic energy to escape the liquid.

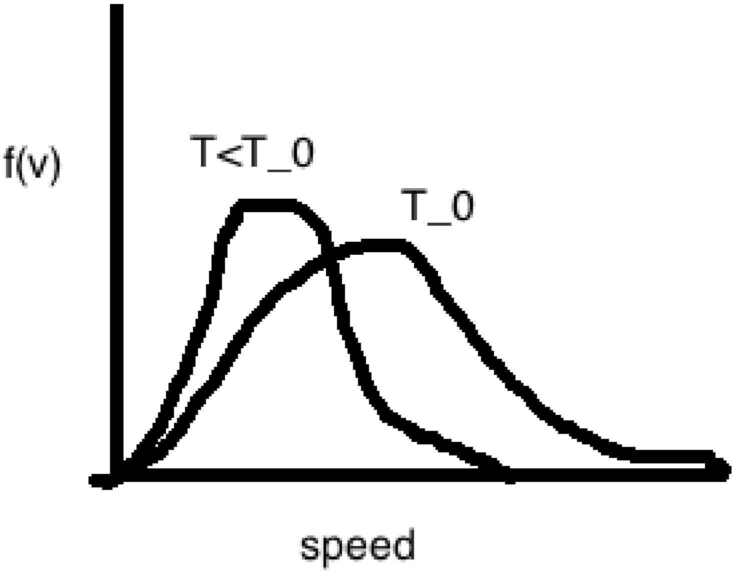
2.2 Part B

Describe and justify the effect of the evaporation process on the temperature of the liquid.

As molecules escape the liquid, they remove energy from the liquid. The average kinetic energy of the molecules decreases accordingly. Temperature, which is proportional to the average kinetic energy of the molecules, must decrease (i.e. the liquid cools). This is the process of evaporative cooling.

2.3 Part C

Make a qualitative drawing of the Maxwell speed distribution of the liquid molecules at T0 and T¡T0.



2.4 Part D

Explain the temperature dependence of the distributions peak position in terms of the microscopic properties of the molecules.

The peak of the distribution occurs at the most probable speed. Translational kinetic energy is given by $KE = 1/2mv^2$. We know that the average energy of a molecule is directly proportional to temperature: $\bar{KE} = 1/2m\bar{v}^2 = 3/2kT$. Thus at lower temperatures we expect the peak of the distribution to occur at a lower speeds.

Problem 1 marked by: Zeinab Jahed

a) 8 marks total

Using: $\beta_{cylinder} = 3 \propto_{AL}$ [1]

Proving: $\beta_{cylinder} = 3 \propto_{AL}$ [4]

$\Delta V_{cylinder} = V_{o\,cylinder} (3 \propto_{AL}) \Delta T$ [2]

$V_{cylinder} = (1 + 3 \propto_{AL} \Delta T) V_{o\,cylinder}$ [1]

$$\boxed{V_{cylinder} = \left(1 + 3\,(25 \times 10^{-6})(T - T_0)\right) V_{o\,cylinder}}$$

b) 3 marks total

$\Delta V_{water} = V_{water} - V_{o\,water} = \beta_{water} V_{o\,water} \Delta T$ [2]

$V_{water} = (1 + \beta_{water} \Delta T) V_{o\,water}$ [1]

$$\boxed{V_{water} = (1 + 210 \times 10^{-6}\,(T - T_0)) V_{o\,water}}$$

c) 4 marks total

the water spills if $\Delta V_{water} > \Delta V_{cylinder}$ [2]

$V_{o\,water} = V_{o\,cylinder}$ and ΔT is same for both. So based on part a) and b) since

$$\beta_{water} > 3 \propto_{AL} \quad [1]$$

$$210 \times 10^{-6} > 3 \times (25 \times 10^{-6})$$

$$\beta_{water} V_{o\,water} \Delta T > (3 \propto_{AL}) V_{o\,cylinder} \Delta T \quad [1]$$

$$\Delta V_{water} > \Delta V_{cylinder}$$

The water expands more than the cylinder and therefore spills out.

d) 5 marks total

to avoid spillage $\Delta V_{water} \le \Delta V_{cylinder}$ [2]

$$\beta_{water} V_{o\,water} \Delta T \le (3 \propto_{AL}) V_{o\,cylinder} \Delta T \quad [1]$$

$$V_{o\,water} = V_{o\,cylinder}$$

$$\beta_{water} \le 3 \propto_{AL} \quad [2]$$

$$\boxed{\propto_{AL} \ge 70 \times 10^{-6}}$$

1 mark was deducted if you just found: $\frac{\beta_{water}}{3} = \propto_{AL}$

Problem 3

a) $\boxed{\text{No, it is not a monatomic ideal gas.}}$ For isovolumetric processes, the first law reads $\Delta E = Q$ because $W = 0$. Furthermore, the equipartition theorem tells us $\Delta E = Q = \frac{d}{2} nR\Delta T$. By definition, this means $C_V = \frac{d}{2} R$ (this relation was also given on the equation sheet). $\boxed{\text{This gas must have } d = 6 \text{ degrees of freedom,}}$ while a monatomic gas would have only three.

b) For an isothermal process, $\boxed{\Delta T = 0}$ and equipartition requires $\boxed{\Delta E = nC_V \Delta T = 0.}$ Thus the first law implies $\boxed{W = Q}$.

c) For an isochoric process $\Delta V = 0$. This implies that $\boxed{W = \int P dV = 0,}$ therefore $\boxed{\Delta E = Q}$. In part (a) we showed that $Q = nC_V \Delta T$ when volume is constant, so

$$\boxed{\Delta T = \frac{Q}{3R \cdot 1\,\text{mole}}}$$

d) For isobaric transformations, $Q = \Delta E + W = nC_V \Delta T + W$ where $W = \int P dV = P\Delta V = nR\Delta T$. Solving these we find

$$\boxed{\Delta T = \frac{Q}{4R \cdot 1\,\text{mole}}}$$

$$\boxed{\Delta E = \frac{C_V}{C_P} Q}$$

Problem 4

a) See Figure 1 for the PV diagram corresponding to this turbine. The heat transfers along the adiabatic legs a-b and c-d are both zero because no heat is transferred, $\boxed{Q_{\text{A-B}} = Q_{\text{C-D}} = 0.}$ Along the isobaric legs, the heat transfer is given by $Q = nC_P\Delta T = n(C_V + R)\Delta T$. Along the leg B-C the heat transfer into the gas is positive

$$\boxed{Q_{\text{B-C}} = n(C_V + R)(T_C - T_B)}$$

For the remaining leg, D-A, the heat is transferred from the gas and Q is a negative quantity,

$$\boxed{Q_{\text{D-A}} = n(C_V + R)(T_A - T_D)}$$

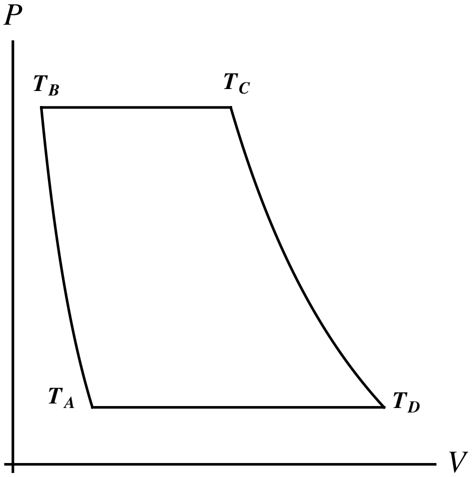


FIGURE 1. A PV diagram for the Brayton cycle.

b) The net work is most easily found by recognizing that $\Delta E = 0$ around the full cycle because the internal energy is a state variable. Then the first law requires

$$\boxed{W_{\text{net}} = \sum Q = Q_{B-C} + Q_{D-A} = n(C_V + R)(T_A - T_B + T_C - T_D)}$$

c) By definition, the efficiency is $e = W/Q_{\text{in}} = 1 - Q_{\text{out}}/Q_{\text{in}}$. We have already calculated these in part (a), but note that $Q_{\text{out}} = -Q_{D-A}$ should be a positive quantity. Then

$$\boxed{e = 1 - \frac{T_D - T_A}{T_C - T_B} = 1 - \frac{T_A}{T_B}}$$

In order to satisfy the second law of thermodynamics, all engines which do work must exhaust some waste heat and the efficiency will always be less than perfect. The most efficient possible engine is the Carnot engine with $e = 1 - T_L/T_H$. In this case, the lowest temperature is T_A but the highest is T_C so the efficiency is less than the Carnot efficiency.

d) For the adiabatic transformations, no heat is transferred and the entropy change is zero: $\boxed{\Delta S_{A-B} = \Delta S_{C-D} = 0.}$ Along the isobars, we can use $dQ = n(C_V + R)\,dT$ and integrate to find the entropy change.

$$\Delta S_{\text{isobaric}} = \int \frac{dQ}{T} = n(C_V + R) \int \frac{dT}{T} = n(C_V + R) \ln \frac{T_f}{T_i}$$

so, also recognizing that the total entropy (or any state variable) change around a closed cycle must be zero $\boxed{(\Delta S_{\text{net}} = 0)}$, we have

$$\boxed{\Delta S_{B-C} = -\Delta S_{D-A} = n(C_V + R) \ln \frac{T_C}{T_B}}$$

We could also calculate $\Delta S_{D-A} = n(C_V + R) \ln T_A/T_D$ directly and use the fact that the total entropy change around the cycle is zero to see that

$$\frac{T_D}{T_A} = \frac{T_C}{T_B}$$

Prob4

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1. 6 point

case: wrong curve for adabatic process: minus 1 point; wrong direction: minus 1 point.

case: use C_v instead of C_p: minus 1 point

2. 4 point

case: $W = -\triangle E$ with a wrong sign with this formula or the final answer e.g. $const * (T_b - T_c) = W_{bc}$. (Both -2, one of them - 1)

case: final answer is not clearly written as an expression of T (variables given): minus 1 point.

3. 5 point

case: wrong Q_{int} e.g. $Q_{int} = Q_{AD}$ or $Q_{BC} + Q_{AD}$: -1 to -2 point

case: without reasoning by $e <= e_{carnot}$: -1 point

4. 5 point

case: wrong integration for $\int \frac{dT}{T}$ e.g. $(T_f - T_i)$, $\int \frac{Q}{T} dT$: -1 to -2 point

case: say $\triangle S_{total} >= 0$ instead of $\triangle S_{total} = 0$: -1 point