PHYSICS 7B, Section 1 – Fall 2013
Midterm 1, C. Bordel
Monday, September 30, 2013
7pm-9pm

Make sure you show your work !

Problem 1 – Thermal expansion (20 pts)

A cylindrical aluminum container of volume V_0 is filled to the top with water at temperature T_0.
The linear and volumetric coefficients of thermal expansion of aluminum and water are given below for the relevant temperature range:
$\alpha_{Al} = 25 \times 10^{-6}\ K^{-1}$; $\beta_{water} = 210 \times 10^{-6}\ K^{-1}$

a) What is the volume of the aluminum container at temperature $T>T_0$?
b) What is the volume of the water at temperature $T>T_0$?
c) Does the water spill out of the container? Explain.
d) What is the condition on the linear thermal expansion coefficient of the container to avoid the spill?

Problem 2 – Maxwell distribution (20 pts)

Molecules in a liquid roughly follow the Maxwell speed distribution.

a) Is it possible for a liquid to vaporize below the boiling point? Explain.
b) Describe and justify the effect of the evaporation process on the temperature of the liquid.
c) Make a qualitative drawing of the Maxwell speed distribution of the liquid molecules at T_0 and $T<T_0$.
d) Explain the temperature dependence of the distribution's peak position in terms of the microscopic properties of the molecules.

Problem 3 – First law (20 pts)

1 mole of an ideal gas with molar specific heat $C_v=3R$ is initially at temperature T_0 and pressure P_0. Vibrational degrees of freedom can be neglected in this temperature range.

a) How many degrees of freedom does the gas have? Could this be a monatomic gas? Explain.

Determine the change in internal energy ΔE, the temperature change ΔT and work W done by the gas when heat Q is added to the gas (b) isothermally, (c) isochorically, (d) isobarically.

Problem 4 - **Second law (20 pts)**

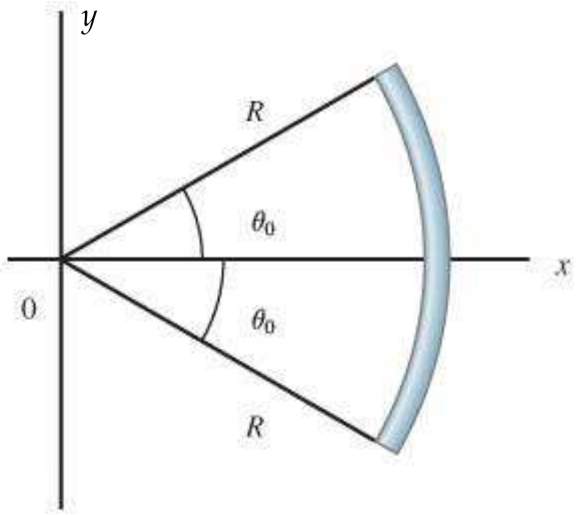
A gas turbine operates under the Brayton cycle, which is an alternate combination of isobaric and adiabatic processes:
AB: adiabatic compression
BC: isobaric expansion
CD: adiabatic expansion
DA: isobaric compression
Assume a monatomic ideal gas of constant volume molar specific heat C_V, undergoing the above cycle defined by the 4 temperatures T_A, T_B, T_C and T_D.

a) Draw the associated (ABCD) cycle on a PV diagram and calculate the heat transferred to or from the gas for every individual process.
b) Calculate the net work done by the gas during a full cycle.
c) Calculate the efficiency of the Brayton engine. Based on your knowledge of the Carnot engine, explain why the efficiency cannot reach 100%.
d) Calculate the entropy change for each process and then the sum for a full cycle. Comment.

Problem 5 – **Electric field (20 pts)**

A thin rod bent into the shape of an arc of a circle of radius R carries a uniform charge per unit length $\lambda>0$. The arc subtends a total angle $2\theta_0$, symmetric about the x axis, as shown below.

a) Determine the magnitude and direction of the electric field at the origin.
b) Use your result from (a) to find the field at the origin when $\theta_0\to0$ but the total charge remains constant. Why would you expect the field to have this form?



$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$f_{Maxwell}(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$E_{int} = \frac{d}{2}NkT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$\Delta E_{int} = Q - W$$

$$W = \int P dV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For an adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2}R$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\overline{g(v)} = \int_0^\infty g(v)\frac{f(v)}{N}dv \ \ (f(v) \text{ a speed distribution})$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$