Final Exam

EE40 - Summer 2014

Gerd Grau

Name:

Discussion section:

Discussion GSI:

Student ID:

Instructions:

Unless otherwise noted on a particular problem, you must show your work in the space provided or on the back of the exam pages.

Underline your answers to each problem with a double line.

Simplify your answers as far as possible unless otherwise noted.

Be sure to provide units where necessary.

GOOD LUCK!

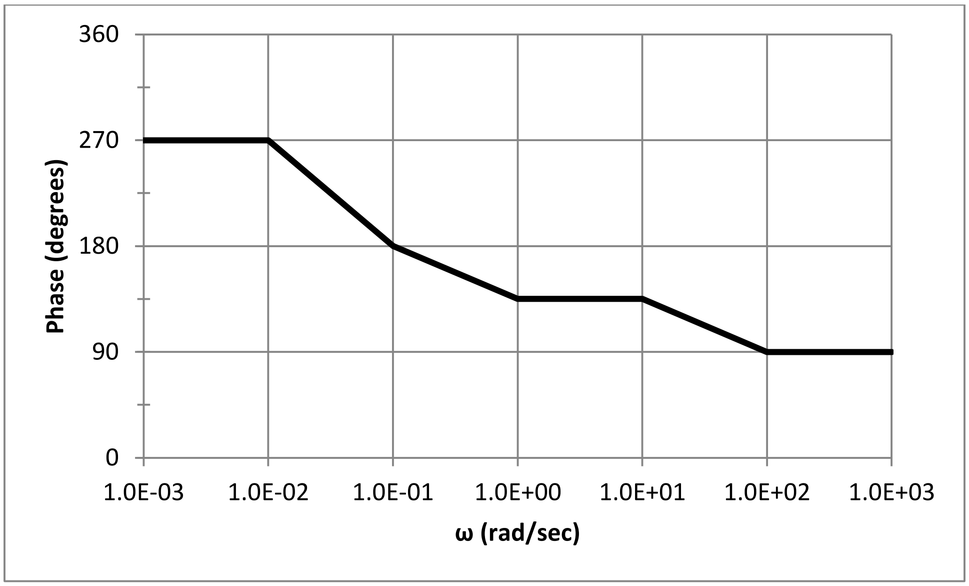
Question	Points	Max
1		13
2		23
3		25
4		29
Total		90

13

Question 1 (13 points):

a) Consider the below Bode phase plot. Write down the simplest transfer function i.e. with the fewest terms that will describe this Bode plot. Use the standard form discussed in lecture e.g. $(1 + j\omega/5)$. Assume the constant pre-factor is +1 and any term including j must also include ω.

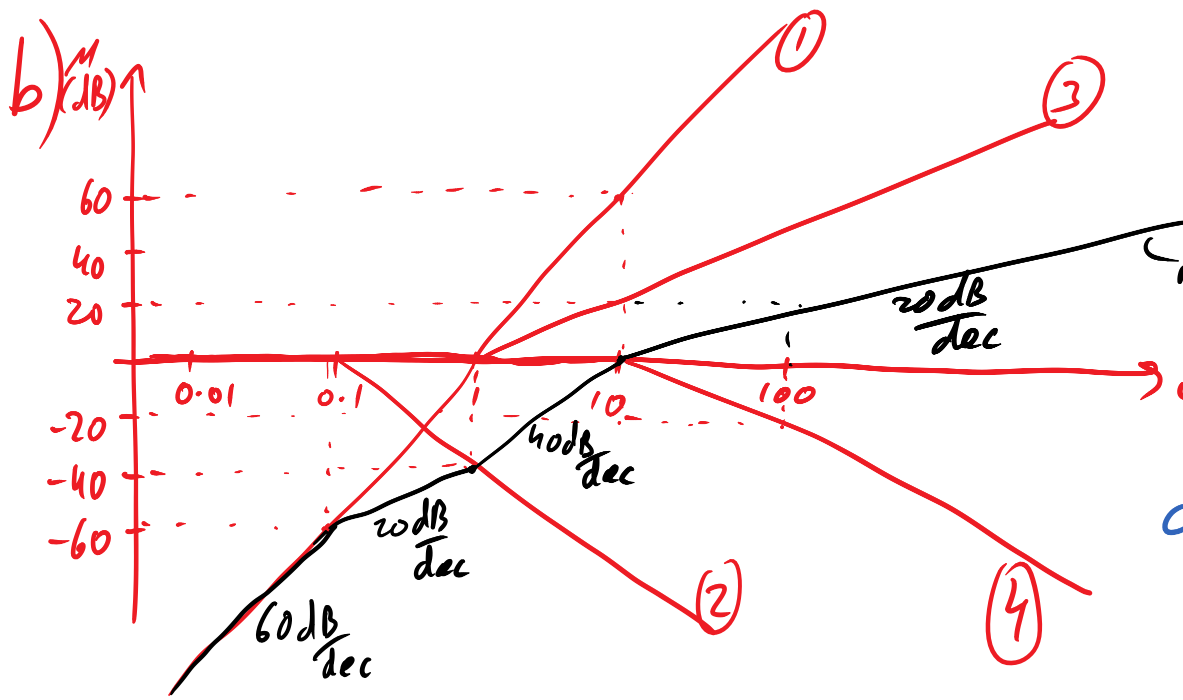
b) Draw the corresponding Bode magnitude plot.



a) $H(\omega) = \dfrac{(j\omega)^3 \left(1+j\frac{\omega}{1}\right)^3}{\left(1+j\frac{\omega}{0.1}\right)^2 \left(1+j\frac{\omega}{10}\right)}$

5

b)



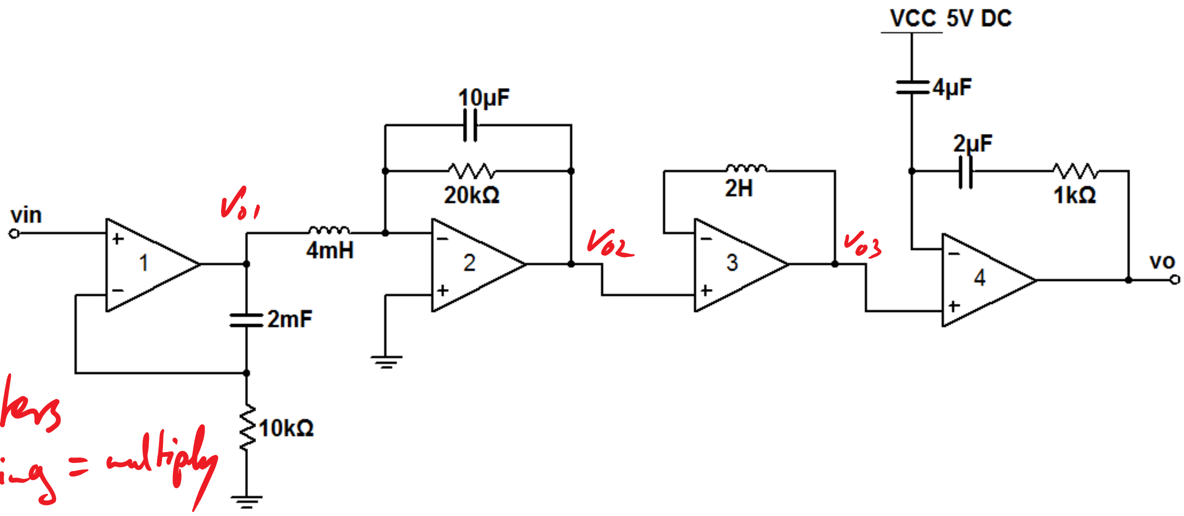
8

don't need to draw individual components if final answer is correct & clear

23

Question 2 (23 points):

a) Consider the below op-amp circuit. Determine the AC transfer function $H(\omega) = v_o/v_{in}$. Put the final result into the standard form. All op-amps are ideal and you can use the negative feedback approximation for all of them.



Active filters
⇒ Cascading = multiply

1: Non-inverting amplifier

$$\frac{V_{o1}}{V_{in}} = 1+\frac{Z_C}{Z_R} = 1+\frac{1}{j\omega CR} = \frac{1+j\omega\times 10k\times 2m}{j\omega\times 10k\times 2m} = \frac{1+j\omega/0.05}{j\omega/0.05}$$

14

2: Inverting amplifier

$$\frac{V_{o2}}{V_{o1}} = -\frac{Z_{11}}{Z_L} = -\frac{\frac{R}{1+Rj\omega C}}{j\omega L} = -\frac{20k}{j\omega 4m\times(1+j\omega\times 20k\times 10\mu)} = -\frac{20k}{j\omega/250\times\left(1+j\frac{\omega}{5}\right)}$$

3: Buffer

$$V_{o3} = V_{o2}$$

4: Non-inverting amplifier (DC acts like AC ground)

$$\frac{V_o}{V_{o3}} = 1+\frac{1k+\frac{1}{j\omega 2\mu}}{\frac{1}{j\omega 4\mu}} = 1+j\omega 1k\times 4\mu+\frac{4}{2} = 3+j\omega/250 = 3\left(1+\frac{j\omega}{250}\times\frac{1}{3}\right) = 3\left(1+j\frac{\omega}{750}\right)$$

$K = 3 \times 2 \times 10^4 \times 250 \times 0.05$

$= 6 \times 10^3 \times 125 = 7.5 \times 10^5$

$$H(\omega) = \frac{V_o}{V_{in}} = \frac{V_o}{V_{o3}} \times \frac{V_{o3}}{V_{o2}} \times \frac{V_{o2}}{V_{o1}} \times \frac{V_{o1}}{V_{in}}$$

$$= \frac{3\left(1+j\frac{\omega}{750}\right) \times 1 \times (-20k) \times (1+j\omega/0.05)}{j\omega/250 \times \left(1+j\frac{\omega}{5}\right) j\omega/0.05} = -\underline{\underline{\frac{7.5 \times 10^5\left(1+j\frac{\omega}{750}\right) \times (1+j\omega/0.05)}{(j\omega)^2\left(1+j\frac{\omega}{5}\right)}}}$$

b) Answer the following questions:

i) Write down the corner frequencies of all simple zeroes in rad/sec (write N/A if none):

750, 0.05

ii) Write down the corner frequencies of all simple poles in rad/sec (write N/A if none):

5

iii) Write down the corner frequencies of all quadratic zeroes in rad/sec (write N/A if none):

N/A

iv) Write down the corner frequencies of all quadratic poles in rad/sec (write N/A if none):

N/A

v) How many zeroes at the origin are there?

0

vi) How many poles at the origin are there?

2

9

vii) What will be the phase for $\omega \rightarrow \infty$

$2 \times (+90°) + 3 \times (-90°) = \underline{-90°}$

viii) What will be the slope of the magnitude Bode plot for $\omega \rightarrow \infty$

$20 \frac{dB}{dec}(+2-3) = \underline{\underline{-20 dB/dec}}$

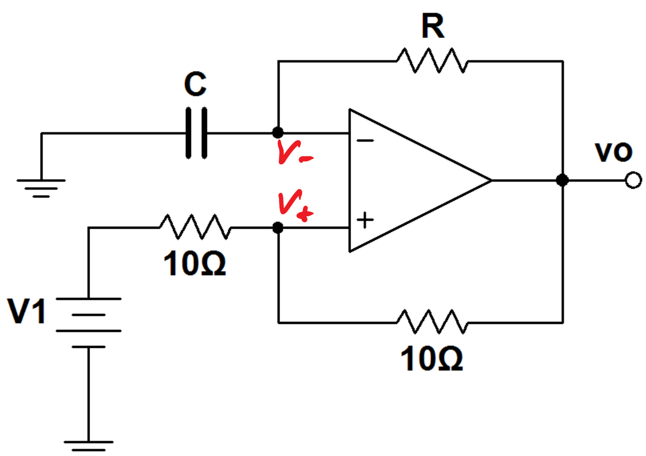
25

Question 3 (25 points):

Consider the below oscillator circuit. You can assume that the op-amp is ideal except for a limited supply voltage of ±5V.

a) Calculate the oscillation period T i.e. the time for one full oscillation cycle as a function of R and C. The voltage V_1 is 0V.
b) Now a DC voltage V_1 is applied by the battery. Calculate the new period T as a function of R, C and V_1.
c) For V_1 = 4V draw the output waveform. Carefully label both axes in terms of V_{supply}, R and C.

If any of your answers contain any terms in the form of e^A or $\ln(A)$ where A is an integer or a fraction of two integers, you can leave it in that form.



a) $v_+ = \frac{v_0}{2}$ (potential divider, ideal op-amp, $V_1 = 0$)

KCL at v_-:

$$C\frac{dv_-}{dt} + \frac{v_- - V_0}{R} = 0$$

Case 1: $V_0 = +5V$

$$v_- = -RC\frac{dv_-}{dt} + V_0$$

$$\Rightarrow v_-(t) = v_-(\infty) + \left(v_-(0) - v_-(\infty)\right)e^{-t/\tau} \qquad \tau = RC$$

During steady state oscillation (i.e. after turn-on)

$V_-(\infty) = V_0 = 5V$

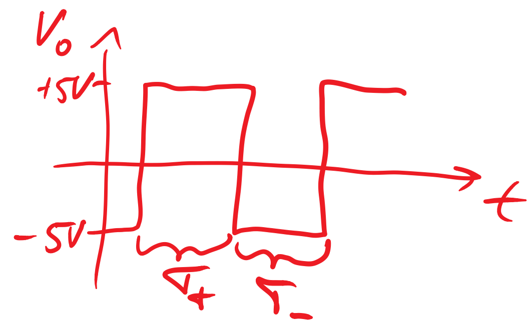
$V_-(0) = -\frac{5}{2}V$ (switching points set by V_+ when $V_0 = -5V$ & $+5V$ respectively)

$V_-(T_+) = +\frac{5}{2}V$

$\Rightarrow \frac{5}{2} = \cancel{5} + \left(-\frac{5}{2} - \cancel{5}\right) e^{-T_+/\tau}$

$-\frac{1}{2} = -\frac{3}{2} e^{-T_+/\tau}$

$\Rightarrow T_+ = \tau \ln(3)$



The same holds for $V_0 = -5V$ (symmetrical)

$\Rightarrow T_- = \tau \ln(3)$

$T = T_+ + T_- = 2\tau \ln(3) = \underline{2RC \ln(3)}$

b) $V_1 \neq 0 \Rightarrow V_+ = \frac{V_0 + V_1}{2}$

For $V_0 = +5V$: $V(T_+) = \frac{V_0 + V_1}{2} = \frac{5+}{2}$

For $V_0 = -5V$: $V(T_-) = \frac{-5 + V_1}{2}$

Case 1: $V_0 = +5V$

$V_-(0) = \frac{-5 + V_1}{2}$

$\frac{5 + V_1}{2} = 5 + \left(\frac{-5 + V_1}{2} - 5\right) e^{-T_+/\tau}$

$$\frac{V_1}{\cancel{2}} - \frac{5}{\cancel{2}} = \left(\frac{V_1}{\cancel{2}} - \frac{3}{\cancel{2}}\cdot 5\right) e^{-T_+/\tau}$$

$$e^{T_+/\tau} = \frac{V_1 - 15}{V_1 - 5}$$

$$T_+ = RC \ln\left(\frac{V_1 - 15}{V_1 - 5}\right)$$

Case 2: $V_0 = -5V$

$$V_-(0) = \frac{5 + V_1}{2}$$

$$\frac{-5 + V_1}{2} = -5 + \left(\frac{5 + V_1}{2} - (-5)\right) e^{-T_-/\tau}$$

$$\frac{V_1}{\cancel{2}} + \frac{5}{\cancel{2}} = \left(\frac{3}{\cancel{2}}\cdot 5 + \frac{V_1}{\cancel{2}}\right) e^{-T_-/\tau}$$

9

$$e^{T_-/\tau} = \frac{V_1 + 15}{V_1 + 5}$$

$$T_- = RC \ln\left(\frac{V_1 + 15}{V_1 + 5}\right)$$

$$T = T_+ + T_-$$

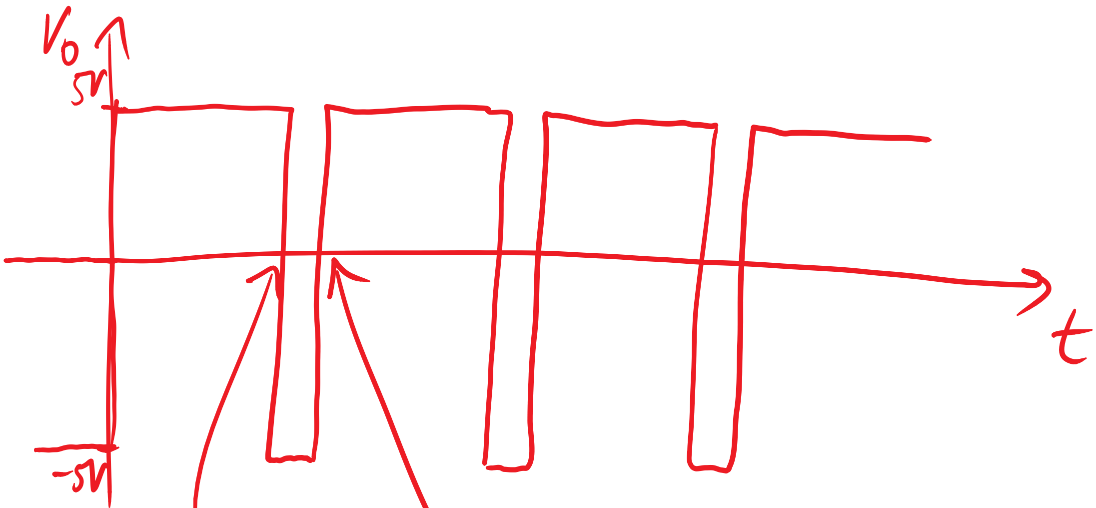
$$= RC \cdot \left(\ln\left(\frac{V_1 - 15}{V_1 - 5}\right) + \ln\left(\frac{V_1 + 15}{V_1 + 5}\right)\right)$$

c) $V_i = 4V$

$$T_+ = RC\ln\left(\frac{4-15}{4-5}\right) = RC\ln\left(\frac{11}{1}\right) \approx 2.4\,RC$$

$$T_- = RC\ln\left(\frac{4+15}{4+5}\right) = RC\ln\left(\frac{19}{9}\right) \approx 0.75RC$$

don't need to calculate exact values if realized effect on duty cycle



$T_+ = RC\ln(11)$

$T_+ + T_- = RC\left(\ln(11) + \ln\left(\frac{19}{9}\right)\right)$

5

29

Question 4 (29 points):

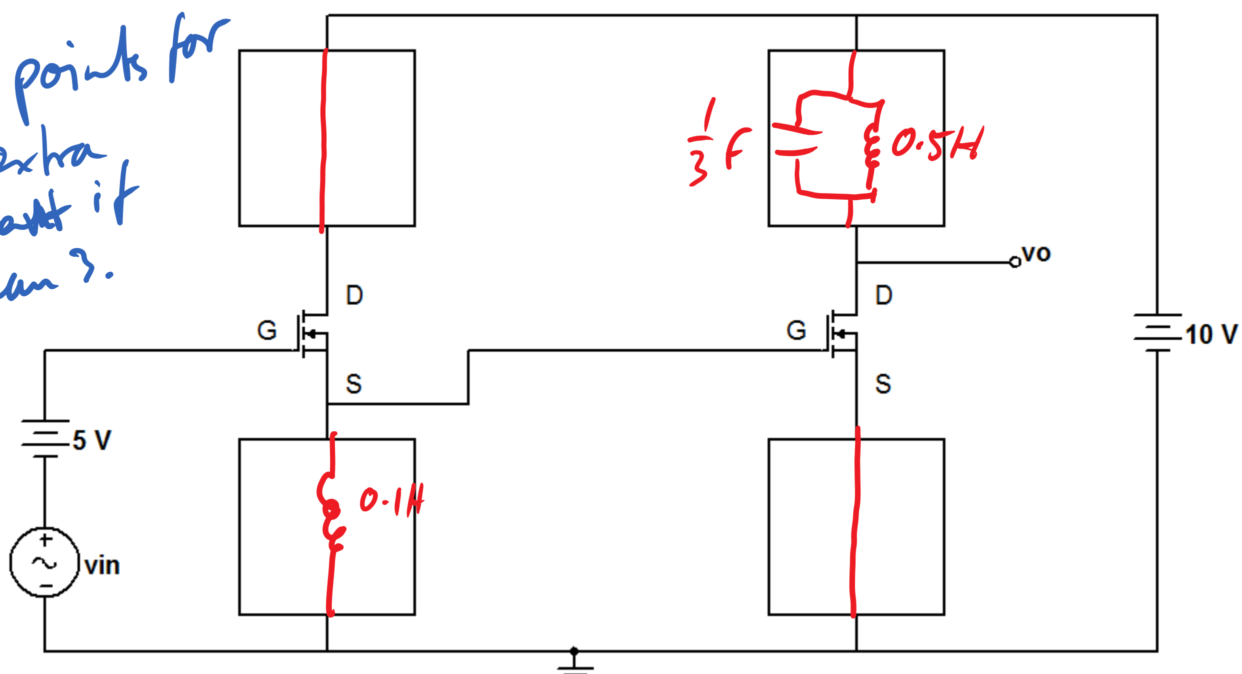
Imagine a customer hires your company to design a circuit that will implement the below transfer function:

$$H(\omega) = -\frac{j\frac{\omega}{100} \times j\frac{\omega}{20}}{\left(1 + j\frac{\omega}{100}\right)\left(1 - \frac{\omega^2}{6}\right)}$$

Due to their current fabrication processes they constrain the types of devices you can use. You are allowed to use two MOSFET transistors and as many capacitors of any capacitance value, inductors of any inductance value and 10kΩ resistors as you need. Your colleague has already started the design process and left you the below circuit drawing before he went on vacation. You need to finish the job by filling in the right impedances. Draw your combinations of R, L, C devices (including values) and simple wires into the blank boxes provided. Clearly justify each choice you make with calculations. Since each additional component translates into extra cost, you will lose points for non-optimum solutions that use a larger number of components than necessary.

You can assume that the applied DC voltages put the MOSFETs into the operating regime that allows you to use the MOSFET small signal equivalent circuit from class. The transconductance of the MOSFETs g_m is 0.1A/V. You can assume that r_d is large enough to be ignored. You can use superposition to ignore the effect of the DC sources. However, you cannot block this DC bias with your impedances i.e. don't use a single capacitor or a capacitor in a purely serial combination of components since that would act as a DC block.

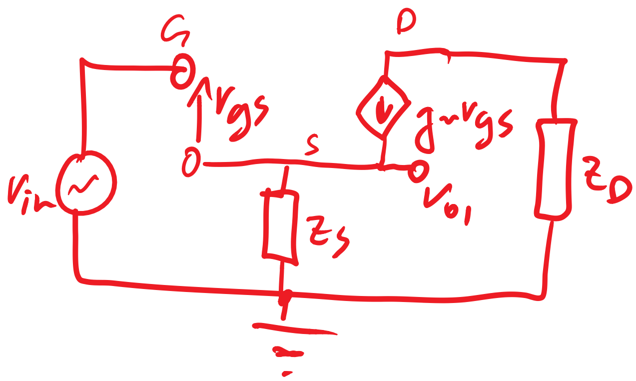
−0.5 points for each extra component if more than 3.



Can analyze MOSFET stages independently since $Z_{in} = \infty$ for ideal MOSFETS.

First stage common drain:

Equivalent circuit:



$$v_{gs} = v_{in} - v_s = v_{in} - v_{o1}$$

$$v_{o1} = g_m v_{gs} Z_S = g_m Z_S (v_{in} - v_{o1})$$

$$\frac{v_{o1}}{v_{in}} = \frac{g_m Z_S}{1 + g_m Z_S}$$

Independent of $Z_D \Rightarrow Z_D$ can be simple wire $(\underline{Z_D = 0})$.

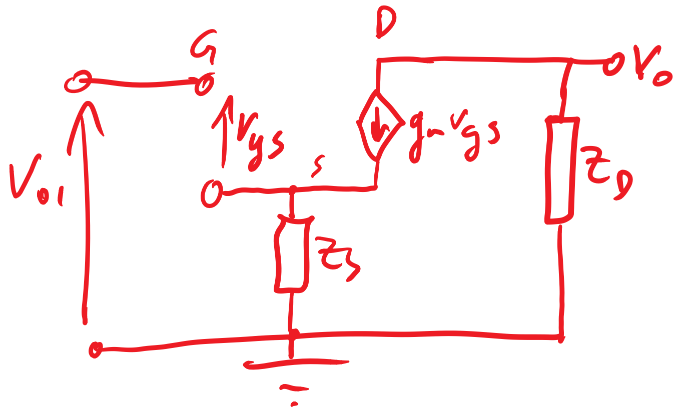
Same cut-off for simple pole and shifted zero at origin.

Looks like $\frac{j\omega/100}{(1+j\omega/100)}$ in desired $H(\omega)$.

$\Rightarrow g_m Z_S = j\omega/100 \Rightarrow Z_S$ is inductor with $j\omega L g_m = j\omega/100$

$$L = \frac{1}{100 \times 0.1} = \underline{0.1\,H}$$

Second stage common source:



$$V_0 = -Z_D g_m V_{gs}$$

$$V_{gs} = V_{01} - g_m V_{gs} Z_S \Rightarrow V_{gs} = \frac{V_{01}}{1+g_m Z_S}$$

$$\frac{V_0}{V_{01}} = -\frac{Z_D g_m}{1+g_m Z_S} = -\frac{j\omega/20}{\left(1-\frac{\omega^2}{6}\right)}$$ (remaining part of $H(\omega)$)

$-\omega^2$ term suggests LC. Can't be in series, has to be parallel:

$$Z_L \| Z_C = \frac{j\omega L \times \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC} = Z_D \qquad g_{m1} Z_D = \frac{j\omega/20}{1-\frac{\omega^2}{6}}$$

$$g_m j\omega L = j\omega/20 \Rightarrow L = \frac{1}{20 \cdot 0.1} = \underline{0.5H}$$

$$1 - \cancel{\omega^2} LC = 1 - \frac{\cancel{\omega^2}}{6} \Rightarrow C = \frac{1}{6L} = \frac{1}{6 \times 0.5} = \underline{\frac{1}{3}F}$$

$$\Rightarrow 1 + g_m Z_S = 1 \Rightarrow Z_S = \underline{0} \text{ (wire)}$$