EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 3
Department of Electrical Engineering and Computer Sciences 27 April 2012
University of California Berkeley

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.

- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.

- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided 8.5" $\times$ 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.

- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.

- Please write neatly and legibly, because *if we can't read it, we can't grade it.*

- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*

- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.

- We hope you do a *fantastic* job on this exam.

MT3.1 (45 Points) Consider the discrete-time LTI filter G whose frequency response is described as

$$\forall \omega \in [-\pi, \pi], \quad G(\omega) = \begin{cases} e^{-i\,\omega\,M/2} & \text{if } 0 \leq |\omega| \leq A \\ 0 & \text{if } A < |\omega| < \pi, \end{cases}$$

where $0 < A < \pi$; M is a non-negative integer; and it is understood that the frequency response replicates with 2π-periodicity outside of the interval specified.

(a) (10 Points) Determine, and provide well-labeled plots of, the magnitude response $|G(\omega)|$ and phase response $\angle G(\omega)$; for this part, assume M is nonzero (and be sure to specify what value of M you use for your plots).

(b) (10 Points) Determine a reasonably simple expression for $g(n)$, the impulse response of the filter.

(c) (10 Points) Show that $g(M-n) = g(n)$, for all integer n. What does this result say about the symmetry of the impulse response (along the n-axis).
Though it's easier to show this symmetry result if you use a correct expression for $g(n)$ from part (b), it's still possible to prove it from the basic properties of the discrete-time Fourier transform.

(d) (15 Points) Consider the discrete-time ideal high-pass filter H whose frequency response is described as

$$\forall \omega \in [0, 2\pi], \quad H(\omega) = \begin{cases} 1 & \text{if } 0 \leq |\omega - \pi| \leq A \\ 0 & \text{if } A < |\omega - \pi| < \pi, \end{cases}$$

where the frequency response is assumed to replicate with 2π-periodicity outside of the interval specified. Provide a well-labeled plot of $H(\omega)$, and determine a reasonably simple expression for $h(n)$, the impulse response of the filter.

MT3.2 (60 Points) Consider a continuous-time p-periodic signal x_c whose Fourier series expansion is

$$x_c(t) = \sum_{m=-\infty}^{+\infty} X_m \, e^{im\omega_0 t},$$

where $\omega_0 = 2\pi/p$ is the fundamental frequency of the signal (and p is the fundamental period). Recall that for this signal $x_c(t+p) = x_c(t)$ for all t.

We sample x_c once every T seconds (where $T > 0$) to create a discrete-time signal x_d as follows:

$$\forall n \in \mathbb{Z}, \quad x_d(n) = x_c(nT).$$

To avoid the trivial case of creating a constant signal x_d, assume, throughout this problem, that the sampling period T is *not* an integer multiple of p.

(a) (20 Points) Prove that the DT signal x_d is periodic if, and only if, the sampling period T is a *rational multiple* of p. In particular, you are to prove that x_d has fundamental period N if, and only if,

$$T = \frac{L}{N} \, p,$$

for some positive integer L. Assume that the ratio L/N is irreducible; in other words, L and N have no common divisors (i.e., they're relatively prime).
Recall that it must be the case that $x_d(n+N) = x_d(n)$ for all integer n.

(b) (20 Points) Assume that the sampling period T is a rational multiple of p. Then, based on part (a), the signal x_d has fundamental period N, a corresponding fundamental frequency $\Omega_0 = 2\pi/N$, and the discrete-time Fourier series (DTFS) expansion

$$x_d(n) = \sum_{k\in\langle N\rangle} \widehat{X}_k \, e^{ik\Omega_0 n}.$$

Determine a reasonably simple expression for $\widehat{X}_k$, the DTFS coefficients of x_d. Your answer should be in terms of the coefficients X_m in the Fourier series expansion of x_c, and, possibly, one or more of the other known parameters.

(c) (20 Points) Now assume that T is *not* a rational multiple of p. Then, based on part (a), x_d is not periodic, so it has no DTFS expansion. Determine a reasonably simple expression for $X_d(\omega)$, the discrete-time Fourier transform of the x_d.

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Problem	Points	Your Score
Name	10	
1	45	
2	60	
Total	**115**	

Potentially useful information:

DTFS:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \quad \longleftrightarrow \quad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n}.$$

CTFS:

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k \, e^{ik\omega_0 t} \quad \longleftrightarrow \quad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) \, e^{-ik\omega_0 t} dt.$$

DTFT:

$$x(n) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\omega) \, e^{i\omega n} d\omega \quad \longleftrightarrow \quad X(\omega) = \sum_{n=-\infty}^{+\infty} x(n) \, e^{-i\omega n}$$

CTFT:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) \, e^{i\omega t} d\omega \quad \longleftrightarrow \quad X(\omega) = \int_{-\infty}^{+\infty} x(t) \, e^{-i\omega t} dt$$

$$\sum_{n=A}^{B} \alpha^n = \begin{cases} B - A + 1 & \text{if } \alpha = 1 \\ \dfrac{\alpha^{B+1} - \alpha^A}{\alpha - 1} & \text{if } \alpha \neq 1. \end{cases}$$