MIDTERMI 2 Tuesday November 4, 2014

Instructor: Prof. A. LANZARA

TOTAL POINTS: 100

TOTAL PROBLEMS: 5

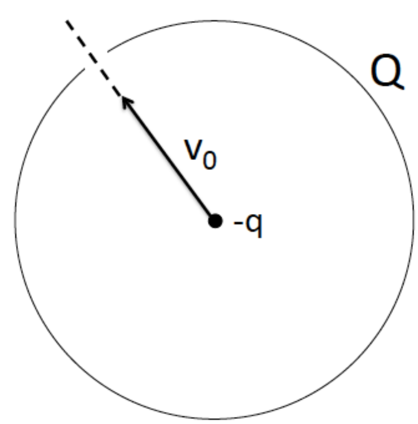
Show all work, and take particular care to explain what you are doing. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make.If you get stuck, skip to the next problem and return to the difficult section later in the exam period.
All answers should be in terms of variables.
Your solutions should show a logical progression of steps. Start with equations on the equation sheet.

Please use one sheet per problem and have them appear in order in your green book. If you need extra room for a problem, place it at the end, after all of the other problems and include a note in the original problem to guide the grader there.

GOOD LUCK!

PROBLEM 1 (Points 20)

A hollow spherical conductor of radius R carries a net positive charge Q.
a) (7pts) Determine the electric field inside and outside the sphere and explain in details how it changes as a function of distance from the center (plot |**E**| vs r diagram).
b) (7pts) Determine the electric potential inside and outside the sphere and explain in details how it changes as a function of distance from the center (plot V vs r diagram).
A small marble of charge (-q) and mass m is launched radially from the center of the sphere toward outside (see figure). Assume there is a tiny hole on the surface of the conductor to allow the charge to pass.
c) (6pts) What is the minimum velocity v_0 that the point charge needs to have to escape from the attraction of the sphere?
Note: You won't be awarded points *for your plots* if the graphs are not physically reasonable.

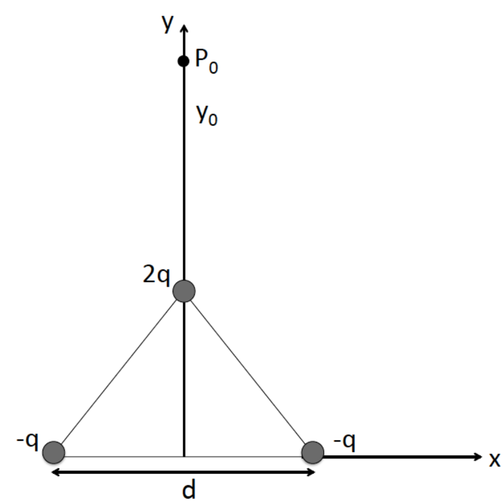


PROBLEM 2 (20pts)

Three charged particles are placed at the corners of an equilateral triangle with side length d as shown in the figure below. The charges are -q, -q and +2q. Find:

a) (8pts) The dipole moment of the three charge system. Draw the net dipole moment.

b) (12pts) The point P_0 is a distance y_0 above the origin on the y-axis as shown in the diagram. Find the electric field due to the three charges at P_0 when P_0 is far away from the charges, or equivalently $y_0 >> d$. You'll need to make an explicit approximation, and you should give your answer to the lowest nonzero order in d/y_0. Make sure you show how you got your result, don't just write down the end formula.

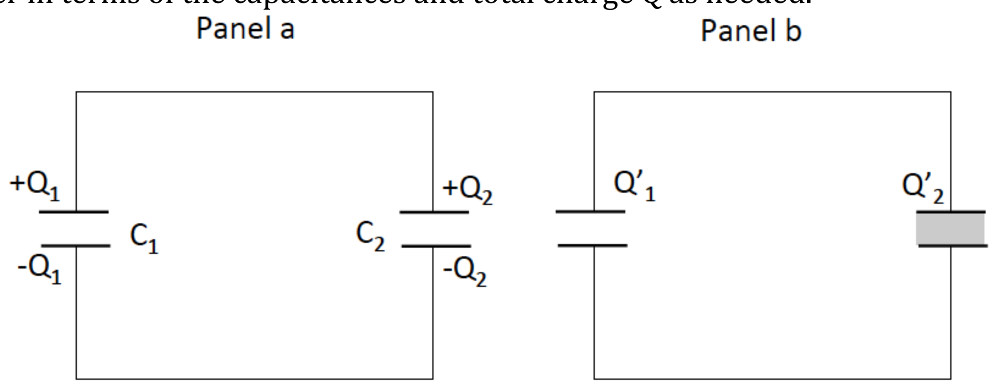


PROBLEM 3 (Points 20)

Two identical capacitors of capacitance C_1 and C_2 (C_1=C_2) are connected in parallel. A total charge Q = Q_1 + Q_2 is distributed among them (see panel a). We now insert a dielectric with dielectric constant *k*, to completely fill the space between the plates of capacitor 2 (see panel b). Determine:

a) (10pts) the final charge on each capacitor.

b) (10pts) the total change of potential energy of the system. Express your answer in terms of the capacitances and total charge Q as needed.



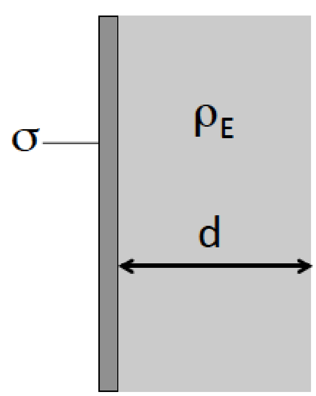
PROBLEM 4 (Points 20)

A very large thin plane has uniform surface charge density σ. Touching it on the right is a long wide slab of insulator with thickness d with uniform volume charge density ρ_E (see figure). Determine the electric field:

a) (5pts) to the left of the plane.

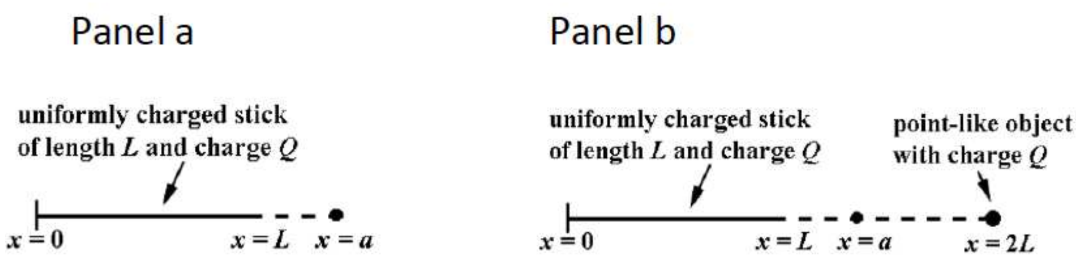
b) (5pts) to the right of the slab.

c) (10pts) everywhere inside the slab.



PROBLEM 5 (20pts)

A stick with length L has a positive charge Q uniformly distributed on it. It lies along the x-axis between the points x=0 and x=L. Express all results in terms of Q, L and fundamental constants.



a) (10pts) Find the electric field of the rod at x=a (see panel a).
We now add a point like object with identical positive charge Q on the x-axis at x=2L, on the right of x=a (see panel b).
b) (6pts) Find the electric field of the point like charge at x=a.
c) (4pts) If x=a is the point where the total electric field is zero, find a.

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} \text{ (point charge)}$$

$$d\vec{E} = \frac{d\lambda}{2\pi\epsilon_0 r}\hat{r} \text{ (line charge)}$$

$$\lambda = \frac{dQ}{ds} \qquad \sigma = \frac{dQ}{dA} \qquad \rho = \frac{dQ}{dV}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$dV = \frac{dQ}{4\pi\epsilon_0 r} \text{ (point charge)}$$

$$dV = -\frac{d\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{r_0}\right) \text{ (line charge)}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$C = \frac{\epsilon A}{d} \text{ (parallel plate)}$$

$$C = \frac{2\pi\epsilon l}{\ln(r_a/r_b)} \text{ (cylindrical)}$$

$$C = 4\pi\epsilon\frac{r_a r_b}{r_a - r_b} \text{ (spherical)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \\ & \text{(Cylindrical Coordinates)} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \\ & \text{(Spherical Coordinates)} \end{aligned}$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\theta) \text{ (law of cosines)}$$