UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits — Spring 2008

MIDTERM EXAMINATION #1

Time allotted: 80 minutes

NAME: ____SOLUTIONS______ ______________________ ______________________
(print) Last First Signature

STUDENT ID#: ____________________

INSTRUCTIONS:

1. **Use the values of physical constants provided below.**
2. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
3. **Clearly mark (underline or box) your answers.**
4. **Specify the units on answers whenever appropriate.**

PHYSICAL CONSTANTS

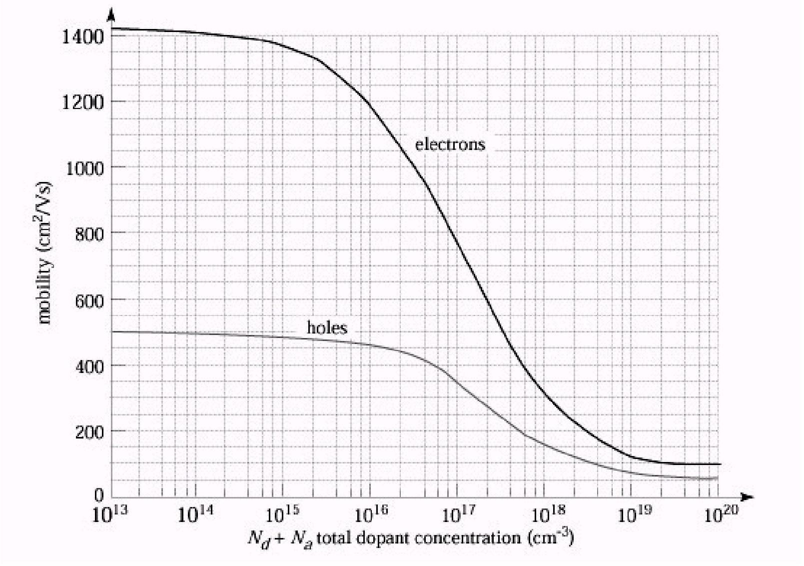
Description	Symbol	Value
Electronic charge	q	1.6×10^{-19} C
Boltzmann's constant	k	8.62×10^{-5} eV/K
Thermal voltage at 300K	$V_T = kT/q$	0.026 V

Note that $\boldsymbol{V_T}$ **ln(10) = 0.060 V** at T=300K

PROPERTIES OF SILICON AT 300K

Description	Symbol	Value
Band gap energy	E_G	1.12 eV
Intrinsic carrier concentration	n_i	10^{10} cm^{-3}
Dielectric permittivity	ε_{Si}	1.0×10^{-12} F/cm

Electron and Hole Mobilities in Silicon at 300K



SCORE: 1 __________ / 25

2 __________ / 25

3 __________ / 30

Total: __________ / 80

Problem 1 [25 points]: Semiconductor Basics

a) A Si resistor is doped with 10^{17} cm^{-3} of phosphorus and 2x10^{17} cm^{-3} of boron impurities.

i) What are the electron and hole concentrations, *n* and *p*, in this sample at room temperature? **[4 pts]**

$$\begin{aligned} N_D &= 10^{17}\ \text{cm}^{-3} \\ N_A &= 2\times10^{17}\ \text{cm}^{-3} \\ p &= N_A - N_D = \boxed{10^{17}\ \text{cm}^{-3}} \\ n &= \frac{n_i^2}{p} = \boxed{10^{3}\ \text{cm}^{-3}} \end{aligned}$$

ii) Estimate the resistivity of this sample. **[5 pts]**

$$\begin{aligned} \rho &= \frac{1}{q\left(n\mu_n + p\mu_p\right)} \\ &\approx \frac{1}{qp\mu_p} \end{aligned}$$

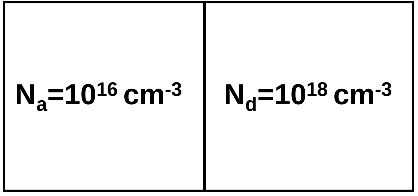
We can find μ_p in the mobility chart using $N_A + N_D = 3\times10^{17}\ \text{cm}^{-3}$. Doing so, we find

$$\begin{aligned} \mu_p &= 250\ \text{cm}^2/\text{V}\cdot\text{s} \\ \rho &= \boxed{0.25\ \Omega\cdot\text{cm}} \end{aligned}$$

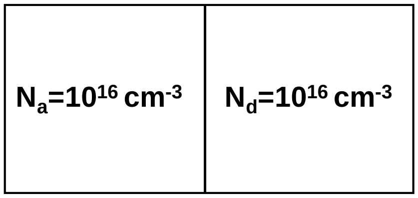
iii) Qualitatively (no calculations required), how would the resistivity change when the temperature goes up to 100ºC? Explain briefly. **[4 pts]**

Increasing the temperature will result in more mobile carriers (i.e., more electrons and holes will have the thermal energy necessary to be conduction electrons/holes), meaning the resistivity will $\boxed{\text{decrease}}$.

b) Consider the two Si p-n junction diode below:



PN Junction A



PN Junction B

i) Find the ratio of the built-in voltages for these two p-n junctions. **[4 pts]**

$$\frac{V_{0A}}{V_{0B}} = \frac{V_T \ln(N_{a,A}N_{d,A}/n_i^2)}{V_T \ln(N_{a,B}N_{d,B}/n_i^2)} = \frac{\log\left(\frac{N_{a,A}N_{d,A}}{n_i^2}\right)}{\log\left(\frac{N_{a,B}N_{d,B}}{n_i^2}\right)} = \frac{\log\left(\frac{10^{34}}{10^{20}}\right)}{\log\left(\frac{10^{32}}{10^{20}}\right)} = \frac{14}{12} = \boxed{\frac{7}{6}}$$

ii) What is the ratio of the current densities under a forward bias voltage of 1V for these two diodes? **[4 pts]**

Assume $\tau_n = \tau_p = \tau$ (given during exam).

$$\frac{J_{D1}}{J_{D2}} = \frac{J_{S1}e^{\frac{V_D}{V_T}}}{J_{S2}e^{\frac{V_D}{V_T}}} = \frac{J_{S1}}{J_{S2}} = \frac{qn_i^2\left(\frac{D_{n,A}}{N_{a,A}L_{n,A}} + \frac{D_{p,A}}{N_{d,A}L_{p,A}}\right)}{qn_i^2\left(\frac{D_{n,B}}{N_{a,B}L_{n,B}} + \frac{D_{p,B}}{N_{d,B}L_{p,B}}\right)}$$

$$= \frac{\frac{D_{n,A}}{N_{a,A}\sqrt{D_{n,A}\tau}} + \frac{D_{p,A}}{N_{d,A}\sqrt{D_{p,A}\tau}}}{\frac{D_{n,B}}{N_{a,B}\sqrt{D_{n,B}\tau}} + \frac{D_{p,B}}{N_{d,B}\sqrt{D_{p,B}\tau}}} = \frac{\frac{\sqrt{D_{n,A}}}{N_{a,A}} + \frac{\sqrt{D_{p,A}}}{N_{d,A}}}{\frac{\sqrt{D_{n,B}}}{N_{a,B}} + \frac{\sqrt{D_{p,B}}}{N_{d,B}}} = \frac{\frac{\sqrt{\mu_{n,A}}}{N_{a,A}} + \frac{\sqrt{\mu_{p,A}}}{N_{d,A}}}{\frac{\sqrt{\mu_{n,B}}}{N_{a,B}} + \frac{\sqrt{\mu_{p,B}}}{N_{d,B}}}$$

We have to look up four mobility values on the given mobility chart. Doing so, we find

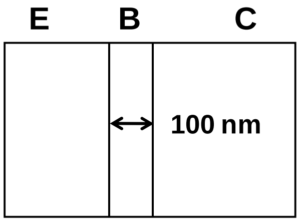
$$\mu_{n,A} = 1200 \text{ cm}^2/\text{V·s}$$
$$\mu_{p,A} = 150 \text{ cm}^2/\text{V·s}$$
$$\mu_{n,B} = 1200 \text{ cm}^2/\text{V·s}$$
$$\mu_{p,B} = 450 \text{ cm}^2/\text{V·s}$$
$$\frac{J_{D1}}{J_{D2}} = \boxed{0.622}$$

iii) Find the ratio of the areal junction capacitances of these two p-n junctions when they are not biased (i.e., 0V). **[4 pts]**

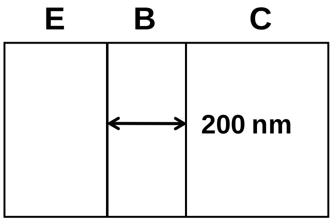
$$\frac{C_{j0,A}}{C_{j0,B}} = \frac{\sqrt{\frac{q\epsilon_{\text{Si}}}{2}\frac{N_{a,A}N_{d,A}}{N_{a,A}+N_{d,A}}\frac{1}{V_{0,A}}}}{\sqrt{\frac{q\epsilon_{\text{Si}}}{2}\frac{N_{a,B}N_{d,B}}{N_{a,B}+N_{d,B}}\frac{1}{V_{0,B}}}} = \sqrt{\frac{N_{a,A}N_{d,A}}{N_{a,B}N_{d,B}} \cdot \frac{N_{a,B}+N_{d,B}}{N_{a,A}+N_{d,A}} \cdot \frac{V_{0,B}}{V_{0,A}}} = \boxed{1.303}$$

<u>Problem 2</u> [25 points]: Bipolar Junction Transistor (BJT)

a) The following two NPN BJTs have the same doping concentrations. The only difference is their base widths: BJT-A has a base width of 100 nm, while BJT-B has a base width of 200 nm. Find the ratio of their current gains. (If you give correct *qualitative* answer, i.e., which BJT has higher current gain and why, you will get half credit). **[6 pts]**



BJT-A



BJT-B

Emitter: N-type, $N_d = 10^{18}\ \text{cm}^{-3}$

Base: P-type, $N_a = 10^{17}\ \text{cm}^{-3}$

Collector: N-type, $N_d = 10^{16}\ \text{cm}^{-3}$

$$\frac{\beta_A}{\beta_B} = \frac{N_{E,A}L_{pE,A}D_{nB,A}}{N_{B,A}W_{B,A}D_{pE,A}} \cdot \frac{N_{B,B}W_{B,B}D_{pE,B}}{N_{E,B}L_{pE,B}D_{nB,B}} = \frac{W_{B,B}}{W_{B,A}} = \boxed{2}$$

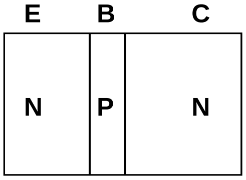
All of the other terms cancel because the doping (and therefore the mobility and diffusion constants) are the same between the two transistors.

b) Consider the following two BJTs. They have identical dimensions and doping profiles, except BJT-A is NPN transistor and BJT-B is PNP transistor. Find the ratio of their current gains. (If you give correct *qualitative* answer, i.e., which BJT has higher current gain and why, you will get half credit). **[6 pts]**

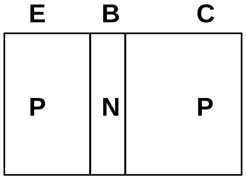
Emitter: N-type, $N_d = 10^{18}\ \text{cm}^{-3}$

Base: P-type, $N_a = 10^{17}\ \text{cm}^{-3}$

Collector: N-type, $N_d = 10^{16}\ \text{cm}^{-3}$



BJT-A



BJT-B

Emitter: P-type, $N_a = 10^{18}\ \text{cm}^{-3}$

Base: N-type, $N_d = 10^{17}\ \text{cm}^{-3}$

Collector: P-type, $N_a = 10^{16}\ \text{cm}^{-3}$

Here, we must again use the assumption that $\tau_n = \tau_p = \tau$.

$$\frac{\beta_A}{\beta_B} = \frac{N_{E,A}L_{pE,A}D_{nB,A}}{N_{B,A}W_{B,A}D_{pE,A}} \cdot \frac{N_{B,B}W_{B,B}D_{nE,B}}{N_{E,B}L_{nE,B}D_{pB,B}} = \frac{N_{E,A}\sqrt{D_{pE,A}\tau}D_{nB,A}}{N_{B,A}W_{B,A}D_{pE,A}} \cdot \frac{N_{B,B}W_{B,B}D_{nE,B}}{N_{E,B}\sqrt{D_{nE,B}\tau}D_{pB,B}}$$

$$\frac{\sqrt{D_{pE,A}}D_{nB,A}}{D_{pE,A}} \cdot \frac{D_{nE,B}}{\sqrt{D_{nE,B}}D_{pB,B}} = \frac{\sqrt{\mu_{pE,A}}\mu_{nB,A}\mu_{nE,B}}{\sqrt{\mu_{nE,B}}\mu_{pE,A}\mu_{pB,B}} = \frac{\sqrt{\mu_{nE,B}}\mu_{nB,A}}{\sqrt{\mu_{pE,A}}\mu_{pB,B}}$$

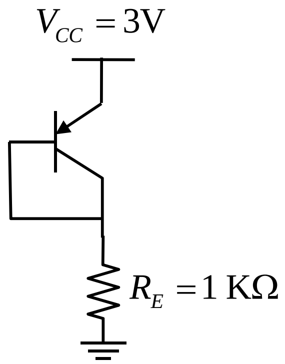
We have to look up all of these mobility values. Doing so, we find

$$\mu_{nE,B} = 300\ \text{cm}^2/\text{V·s}$$
$$\mu_{nB,A} = 750\ \text{cm}^2/\text{V·s}$$
$$\mu_{pE,A} = 150\ \text{cm}^2/\text{V·s}$$
$$\mu_{pE,B} = 350\ \text{cm}^2/\text{V·s}$$
$$\frac{\beta_A}{\beta_B} = \boxed{3.030}$$

c) Answer this question *qualitatively*. For the two BJTs in Part a), which BJT will have larger Early voltage? Why? **[4 pts]**

Device $\boxed{\text{B}}$ will have a larger Early voltage (i.e., it will suffer less from the Early effect than device A). This is because the base width in B is larger than it is in A. This means that any change in the base width due to a change in the reverse bias on the base-collector junction will result in a smaller relative change in the base width in B than in A.

d) Solve the bias point of the following PNP transistor (I_C, V_{EB}, V_{EC}). Assume $I_S = 10^{17}$ A, β=100, and $V_A = \infty$ **[5 pts]**



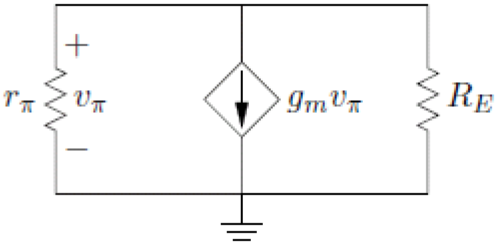
Assume $I_S = 10^{-17}$ A (correction made during exam).

$$V_{CC} - V_{EB} = I_E R_E = \frac{1+\beta}{\beta} I_C R_E$$

$$V_{EB} = V_{CC} - \frac{1+\beta}{\beta} I_C R_E = V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$I_C = \boxed{2.12\ \text{mA}}$$

e) Draw the small-signal model of the circuit in Part d). Specify all the small signal parameters used (e.g., g_m, r_π, etc). **[4 pts]**



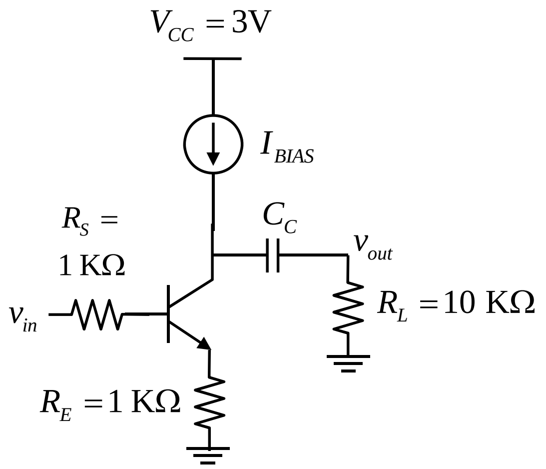
$$g_m = \frac{I_C}{V_T} = \boxed{81.5\ \text{mS}}$$

$$r_\pi = \frac{\beta}{g_m} = \boxed{1.226\ \text{k}\Omega}$$

<u>Problem 3</u> [30 points]: BJT Amplifiers

a) Consider the BJT amplifier shown below with $I_{BIAS} = 1$ mA.

Assume $I_S = 10^{17}$ A, β=100, and $V_A = 10V$.



i) Find the value of V_{BE}. **[4 pts]**

Assume $V_C = 2$ V (given during exam) and $I_S = 10^{-17}$ A (correction made during exam).

$$V_C = 2 \text{ V}$$
$$I_C = 1 \text{ mA (fixed by the current source)}$$
$$V_E = I_E R_E = \frac{1+\beta}{\beta} I_C R_E = 1.01 \text{ V}$$
$$V_{CE} = 0.99 \text{ V}$$
$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE}}{V_A}\right)$$
$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S\left(1 + \frac{V_{CE}}{V_A}\right)}\right) = \boxed{836 \text{ mV}}$$

ii) Is the BJT in the active mode? Why? **[4 pts]**

Yes. The base-emitter junction is forward biased and the base-collector junction is reverse biased (i.e., $V_{CE} > V_{BE}$).

iii) Find the small signal parameters of the BJT under this bias condition. **[4 pts]**

$$g_m = \frac{I_C}{V_T} = \frac{1}{26} = \boxed{38.5 \text{ mS}}$$
$$r_\pi = \frac{\beta}{g_m} = \boxed{2.6 \text{ k}\Omega}$$
$$r_o = \frac{V_A}{I_C} = \boxed{10 \text{ k}\Omega}$$

iv) What is the expression for the voltage gain? What is its numerical value? **[6 pts]**

For this part, assume $V_A = \infty$ (given during exam).

$$A_v = \boxed{-\frac{R_L}{\frac{1}{g_m} + R_E + \frac{R_S}{1+\beta}}} = \boxed{-9.65}$$

You could also write the gain as the gain from the input to the base times the gain from the base to the collector, which would look like:

$$A_v = -\frac{r_\pi + (1+\beta)R_E}{R_S + r_\pi + (1+\beta)R_E} \cdot \frac{R_L}{\frac{1}{g_m} + R_E} = -9.65$$

v) What is the expression for the input impedance (seen by v_{in})? What is its numerical value? **[6 pts]**

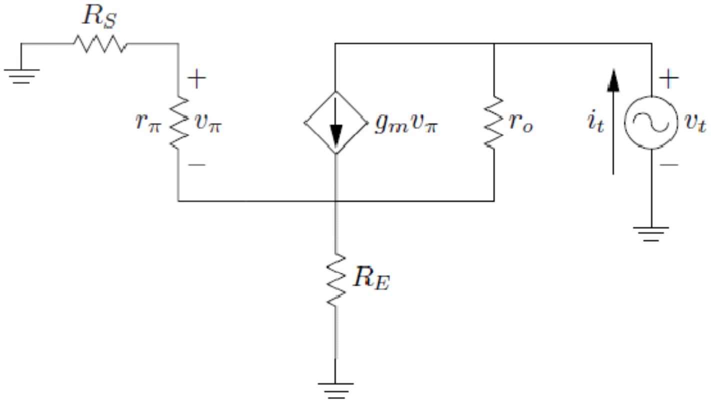
For this part, assume $V_A = \infty$ (given during exam).

$$R_{in} = \boxed{R_S + r_\pi + (1+\beta)R_E} = \boxed{104.6\ \mathrm{k\Omega}}$$

vi) What is the expression for the output impedance (seen by v_{out})? What is its numerical value? **[6 pts]**

For this circuit, we must perform small-signal analysis to determine the output resistance. The derivation is similar to the derivation of the output impedance of a common emitter amplifier with emitter degeneration.

Here's the small-signal model with a test source at the output (excluding R_L, which obviously goes in parallel with the result from this analysis):



$$\frac{v_t - v_e}{r_o} + g_m v_\pi = i_t$$
$$i_t[R_E \parallel (r_\pi + R_S)] = v_e$$
$$v_\pi = -v_e \frac{r_\pi}{r_\pi + R_S}$$

$$\begin{aligned} v_e &= v_t + g_m r_o v_\pi - i_t r_o \\ &= v_t - g_m r_o v_e \frac{r_\pi}{r_\pi + R_S} - i_t r_o \end{aligned}$$
$$v_e = \frac{v_t - i_t r_o}{1 + g_m r_o \frac{r_\pi}{r_\pi + R_S}}$$

$$i_t[R_E \parallel (r_\pi + R_S)] = \frac{v_t - i_t r_o}{1 + g_m r_o \frac{r_\pi}{r_\pi + R_S}}$$
$$i_t\left\{[R_E \parallel (r_\pi + R_S)] + \frac{r_o}{1 + g_m r_o \frac{r_\pi}{r_\pi + R_S}}\right\} = \frac{v_t}{1 + g_m r_o \frac{r_\pi}{r_\pi + R_S}}$$
$$R_{out} = R_L \parallel \frac{v_t}{i_t} = \boxed{R_L \parallel \left\{r_o + \left(1 + g_m r_o \frac{r_\pi}{r_\pi + R_S}\right)[R_E \parallel (r_\pi + R_S)]\right\}} = \boxed{9.58\ \text{k}\Omega}$$