Name Key

SID

BioE 102 Fall 2013
Midterm #2

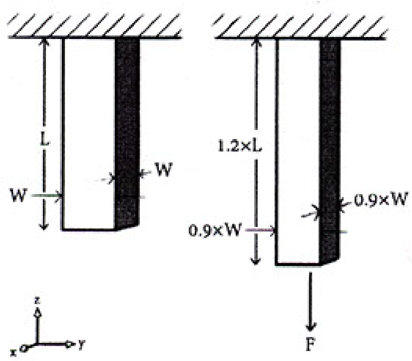
Instructions: Please write legibly; write your name and SID on the upper right corner of each page.

1. **Hook's Law**

Consider the transverse isotropic beam shown below. Unloaded, the beam has a length, L and width W. In tension under the force F, the beam is stretched to a length of 1.2×L and width 0.9×W (represented in the diagram below). **A.** Calculate all normal strains and use these to calculate the relevant Poisson's ratio. **B.** Solve for the σ_{zz} in terms of F and W and use this value to solve for the relevant elastic modulus.

You may assume linear elastic behavior

(*Hint: At equilibrium, the area that the force, F, acts over may be different than the area at rest.*)



A| $\varepsilon_{zz} = \frac{\Delta l_z}{l_{0z}} = \frac{0.2\,L}{L} = 0.2$ (7pt.)

$\varepsilon_{yy} = \frac{\Delta l_y}{l_{0y}} = \frac{-0.1\,W}{W} = -0.1 = \varepsilon_{xx}$ (9 pt)

$\nu = \frac{-\varepsilon_{lateral}}{\varepsilon_{axial}} = \frac{0.1}{0.2} = \frac{1}{2}$ (7 pt)

B| $\sigma_{zz} = \frac{F}{A} = \frac{F}{(0.9W)^2} = \frac{F}{0.81\,W^2}$ (5 pt)

$\sigma_{zz} = E\,\varepsilon_{zz}$

$E = \frac{\sigma_{zz}}{\varepsilon_{zz}} = \frac{F}{.162\,W^2} = 6.17\,\frac{F}{W^2}$ (5pt)

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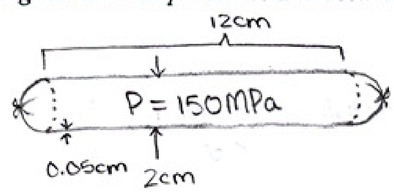
2. **Pressure Vessels**

You're grilling a hotdog and the heat from the grill is causing the internal gasses to expand, creating an internal pressure. You can model the hotdog as a 12cm long cylinder with a diameter of 2cm and two spherical caps with a casing that is a mere 0.05cm thick. A fully cooked hotdog will have an internal pressure of 3.2MPa. **A.** Calculate the normal stresses on the surface of the hotdog on both the spherical ends and the cylindrical center. **B.** The hotdog will break open if a normal stress surpasses 150MPa or a shear stress surpasses 26MPa. Determine if the hotdog will break open before it is fully cooked.

You may neglect external pressures and assume linear elastic behavior.



$\frac{h}{a} = \frac{0.05cm}{1cm} = \frac{1}{20} \Rightarrow$ thin wall approximation

(3pt/ea. × 6)

A| $\sigma_{\theta\theta,\,cylinder} = \frac{Pa}{h} = \frac{3.2MPa\left(\frac{2cm}{2}\right)}{0.05cm} = 64MPa$

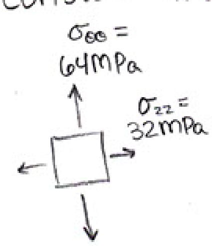
$\sigma_{\theta\theta,\,sphere} = \sigma_{\phi\phi,\,sphere} = \sigma_{zz,\,sphere} = \frac{Pa}{2h} = \frac{3.2MPa\left(\frac{2cm}{2}\right)}{2\cdot 0.05cm} = 32MPa$

$\sigma_{rr,\,cylinder} = \sigma_{rr,\,sphere} \cong -\frac{P}{2} = \frac{3.2MPa}{2} = 1.6MPa$

$\sigma_{rr,\,cylinder}$ & $\sigma_{rr,\,sphere}$ are negligible

B| largest stress on cylindrical portion:
consider the following FBD:

(15pt)



$\sigma_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$

$= \frac{32+64}{2} \pm \sqrt{\left(\frac{32-64}{2}\right)^2 + 0} = 64MPa,\ 32MPa$

$\Rightarrow$ no normal stress will ever be greater than 150MPa!

$\tau_m = \pm\sqrt{\left(\frac{32-64}{2}\right)^2} = 16MPa$

$\Rightarrow$ no shear stress will ever be greater than 26MPa!

Woohoo! fully cooked hotdogs for everyone!

(Full points for considering radial shear stress)

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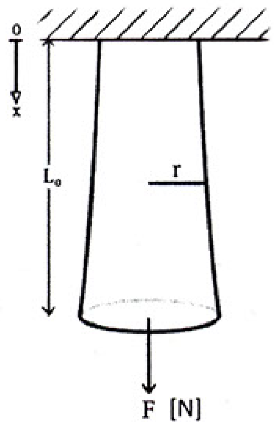
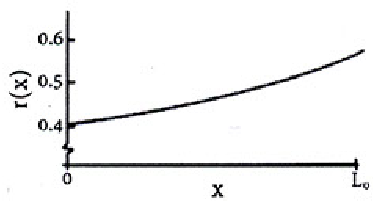
3. Extension

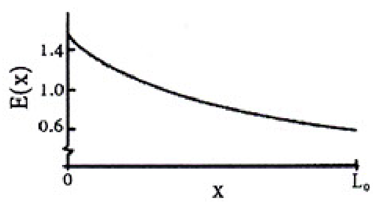
A force is applied on a beam with circular cross-section as shown below. Both the radius and the Young's modulus of the beam are functions of x. **A.** What is the total deformation, δ, of the beam under the applied force? **B.** If you were to use strain gauges to measure the strain along this beam, at what location x would you measure the largest strain in the x direction?

You may neglect any strain due to Poisson's ratio and assume linear elastic behavior





$$r(x) = \sqrt{\frac{1}{\left(2-\frac{x}{L_0}\right)\pi}}\; r_0 \quad [\text{m}]$$



$$E(x) = \frac{1}{\left(\frac{x}{L_0}+\frac{2}{3}\right)} E_0 \quad [\text{Pa}]$$

A] $\delta = u_x(L_o) = \int_0^{L_o} \frac{f(x)}{A(x)E(x)}\,dx$ (5pt) | (17 pt)

(5 pt)

$\because A(x) = \pi r(x)^2 = r_o^2\left(\frac{1}{2-\frac{x}{L_o}}\right)$

$= \frac{F}{r_o^2 E_o}\int_0^{L_o} \frac{1}{\left(\frac{1}{2-\frac{x}{L_o}}\right)\left(\frac{1}{\frac{x}{L_o}+\frac{2}{3}}\right)}\,dx = \frac{F}{r_o^2 E_o}\int_0^{L_o}\left(2-\frac{x}{L_o}\right)\left(\frac{x}{L_o}+\frac{2}{3}\right)dx$

$= \frac{F}{r_o^2 E_o}\int_0^{L_o}\left(\frac{-1}{L_o^2}x^2 + \frac{4}{3L_o}x + \frac{4}{3}\right)dx$

$= \frac{F}{r_o^2 E_o}\left(\frac{-1}{3L_o^2}x^3 + \frac{2}{3L_o}x^2 + \frac{4}{3}x\right)\Big|_0^{L_o} = \frac{F}{r_o^2 E_o}\left(\frac{-L_o^3}{3L_o} + \frac{2L_o^2}{3L_o} + \frac{4L_o}{3}\right)$ (7 pt)

$= \frac{5FL_o}{3r_o^2 E_o}$

B] $\varepsilon(x) = \frac{d}{dx}\left(\frac{f(x)}{A(x)E(x)}\right)$ (10 pt) | (17 pt)

$= \frac{F}{r_o^2 E_o}\frac{d}{dx}\left(\frac{-1}{L_o^2}x^2 + \frac{4}{3L_o}x + \frac{4}{3}\right)$

$= \frac{F}{r_o^2 E_o}\left(\frac{-2}{L_o^2}x + \frac{4}{3L_o}\right)$ $\because$ maximum when $\frac{d}{dx} = 0$

$\Rightarrow \frac{-2}{L_o^2}x + \frac{4}{3L_o} = 0$ (7 pt)

$x = \frac{2}{3}L_o$