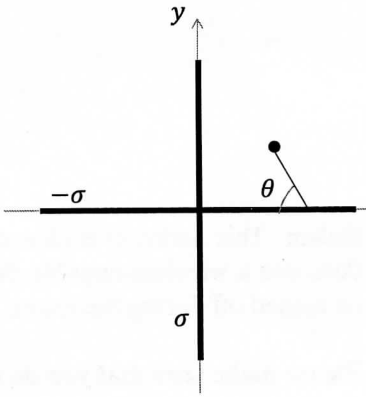
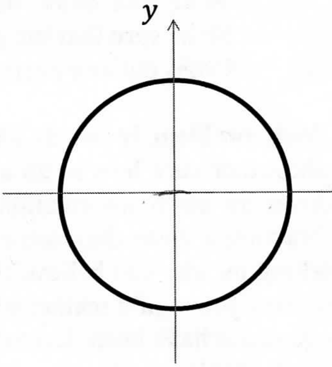
1. Two infinite sheets of charge, one with charge density, σ, the other with charge density, $-\sigma$, are oriented at 90° from one another (see figure on right). A point charge, $q < 0$, with mass, m, is attached to the bottom sheet with a string and floats above that sheet at an angle, θ.



   a. What are the components of the electric field in each of the four quadrants?
   b. What is θ? Express it in terms of σ, q, m, ϵ_0, and the acceleration of gravity, g.

2. The figure on the right shows a ring of charge with radius, a, and has a linear charge density,

$$\lambda(\theta) = \lambda_0 \cos(k\theta)$$



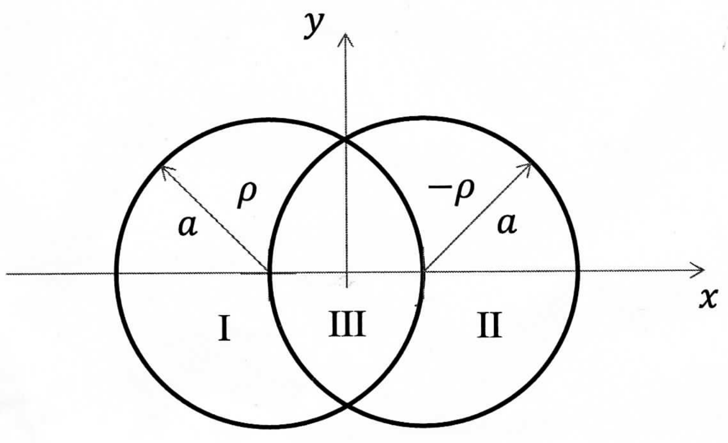
where k is a constant and θ is measured counter-clockwise from the positive x −axis. You may need the following identities:

$$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$$
$$\cos a \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$$

   a. What is the electric field, E_x and E_y, at the center of the circle?
   b. For what values of k is $E_x = 0$? For what values of k is $E_y = 0$? (It is possible to answer this part even if you were not able to complete part a.)

3. The figure below shows two intersecting **spheres**, each of radius, a. The far edge of one sphere passes through the center of the other sphere. Region I is filled with an insulator that has a charge density, ρ. Region II is filled with an insulator that has a charge density, $-\rho$. Region III is filled with an insulator with no **net** charge.

   a. Find the electric field on the x −axis when $-a \le x \le a$. Express it in terms of ρ, a, and ϵ_0 (yes, x is absent from this list).
   b. Find the electric field on the positive x −axis when $x \gg a$. Make approximations and keep terms to the lowest nontrivial order in x/a.



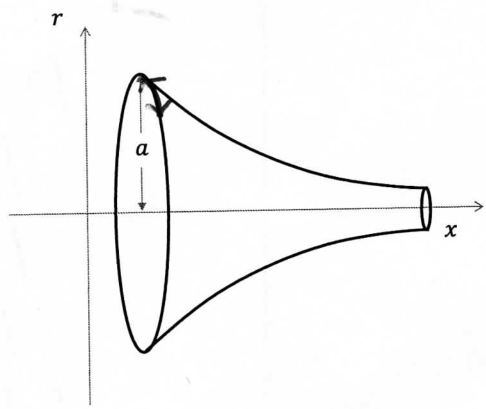
4. The figure below shows insulating surface that has a surface charge, $\sigma > 0$. The profile of the surface is described by the equation,

$$r = \frac{a^2}{x}.$$

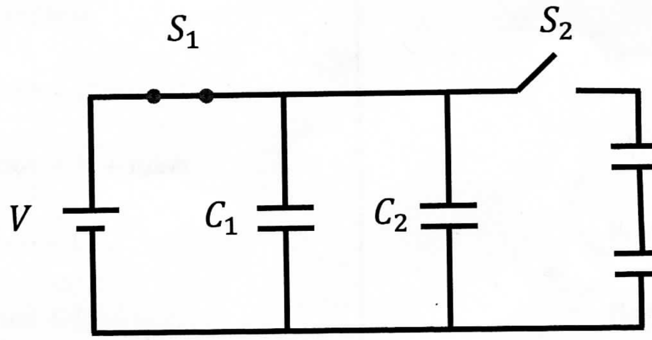
The surface starts at $x = a$ and extends out to $x \to \infty$. Remember that the infinitesimal arc-length, ds, along the edge of the cone is given by

$$ds = [(dr)^2 + (dx)^2]^{1/2} = \left[\left(\frac{dr}{dx}\right)^2 + 1\right]^{1/2} dx = \left[1 + \left(\frac{dx}{dr}\right)^2\right]^{1/2} dr$$

a. **Using the slice-and-dice method**, find the electric potential, $V(0)$, at the origin. (You will get the great majority of the points for this part of the problem by setting up the integral in terms of one integration variable and with the correct limits in integration.)
b. A charge, $q < 0$, with mass, m, is placed at the origin. What is the velocity, v_0, along the $+x$-axis that must be given to the charge so that it just reaches $+\infty$? You may express your answer in terms of $q, v_0, m,$ and $V(0)$.



5. The figure to the right shows a capacitor network made out of four capacitors. Initially, the switch S_1 is closed, and $C_1 = 1.0\ F$ and $C_2 = 2.0\ F$ are connected to a voltage source, $V = 3.0\ V$. Switch S_1 is then opened, and S_2 closed so that C_1 and C_2 are connected to $C_3 = 1.0\ F$ and $C_4 = 2.0\ F$. After all the charges have stopped moving, find the charges on all four capacitors.



$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} \text{ (point charge)}$$

$$d\vec{E} = \frac{d\lambda}{2\pi\epsilon_0 r}\hat{r} \text{ (line charge)}$$

$$\lambda = \frac{dQ}{ds} \qquad \sigma = \frac{dQ}{dA} \qquad \rho = \frac{dQ}{dV}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$dV = \frac{dQ}{4\pi\epsilon_0 r} \text{ (point charge)}$$

$$dV = -\frac{d\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{r_0}\right) \text{ (line charge)}$$

$$\vec{E} = -\vec{\nabla}V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$C = \frac{\epsilon A}{d} \text{ (parallel plate)}$$

$$C = \frac{2\pi\epsilon l}{\ln(r_a/r_b)} \text{ (cylindrical)}$$

$$C = 4\pi\epsilon\frac{r_a r_b}{r_a - r_b} \text{ (spherical)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{}$$

$$\begin{aligned} \vec{\nabla}f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \end{aligned}$$

(Cylindrical Coordinates)

$$\begin{aligned} \vec{\nabla}f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \end{aligned}$$

(Spherical Coordinates)

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\int (1+x^2)^{-1/2}dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1}dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2}dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2}dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)}dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)}dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\theta) \text{ (law of cosines)}$$