Physics 8B

Spring '14

Midterm 1 solutions

1) a. negative. Since the sheet of charge exerts a force on q_1 in the positive *x* direction, q_2 must exert a force in the negative *x* direction to cancel it. Hence, q_2 must be negative to attract q_1.

b.

$F_1(between\ sheet\ and\ q_1) = \frac{\sigma q_1}{2\varepsilon_0}$

$$F_2(between\ q_1 and\ q_2) = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

Calculate the *x*-component of F_2 and substitute appropriate values. Then set this expression equal to F_1, which also points in the *x* direction:

$$F_{2,x} = \frac{q_1 q_2}{4\pi\varepsilon_0 (5)^2}\left(\frac{4}{5}\right) = \frac{\sigma q_1}{2\varepsilon_0} = \left(\frac{2}{25\pi}\right)\frac{q_1}{2\varepsilon_0}$$

Solving for q_2 yields $q_2 = -5C.$

2) a. $I_1 + I_2 - I_3 = 0$

b.

$V - I_1 R - I_3 R = 0$

$2V - I_2 R - I_3 R = 0$

c. Solving the above 3 equations yields $I_1 = 0$

3) a. Due to symmetry, the electric field from two charges that are diametrically opposed will cancel. Thus, the only contribution comes from the charge lying on the x-axis.

b. If a charge were moved from $+\infty$ to the origin, it will be a distance *d* away from any charge.

$\Delta V = -\int_{+\infty}^{d} \frac{k(5q)}{r^2} = 5kq\left(\frac{1}{d} - \frac{1}{\infty}\right) = \frac{5kq}{d}$ when all charges are considered.

c. $U = VQ = \frac{1}{2}mv^2$

$v = \sqrt{\frac{10kqQ}{dm}}$

4) a. $q_+ = \frac{4}{3}\pi a^3 \rho$

$q_- = \frac{4}{3}\pi(b^3 - a^3)(-\rho)$

$q_+ + q_- = 0$

$\frac{4}{3}\pi a^3 \rho = \frac{4}{3}\pi(b^3 - a^3)(-\rho)$

$\frac{b^3}{a^3} = 2$

b. $E_I \cdot A = \frac{q_{enclosed}}{\varepsilon_0}$

$E_I \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi r^3 \rho}{\varepsilon_0}$

$E_I = \frac{r\rho}{3\varepsilon_0}$

c.

$E_{II} \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi a^3 \rho - \frac{4}{3}\pi(r^3 - a^3)\rho}{\varepsilon_0}$

$E_{II} = -\frac{r\rho}{3\varepsilon_0} + \frac{2a^3\rho}{3r^2}$

5. Since the charge starts from rest and ends at rest, the change in potential energy with respect to the left wire must exactly balance the change with respect to the right wire.

$E(line\ of\ charge) = \frac{2k\lambda}{r}$

$\Delta V = 0 = \Delta V_L + \Delta V_R$

$\Delta V = -\int_a^b \frac{2k\lambda}{r}$

Substituting the initial and final positions of the charge with respect to the left and right wire yields the following:

$\Delta V_L + \Delta V_R = \frac{\lambda_L}{2\pi\varepsilon_0} ln\left(\frac{d/4}{7d/8}\right) + \frac{\lambda_R}{2\pi\varepsilon_0} ln\left(\frac{3d/4}{d/8}\right) = 0$

$\frac{\lambda_R}{\lambda_L} = -\frac{-ln(2/7)}{\ln(6)}$

A = 2/7, B = 6