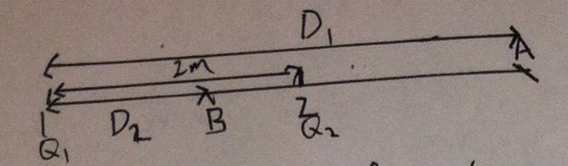
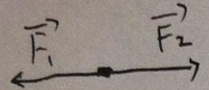
Sol. to Q1.



a) The force diagram for charge q @ point A is like the following



where F_1 is the force bet. Q_1 & q, F_2 is bet Q_2 & q.

q is at stationary state, hence we have

$\vec{F_1} + \vec{F_2} = 0$ *) (5 pts)

By coulomb's law. $\vec{F_1} = \frac{kQ_1q}{D_1^2}$, $F_2 = \frac{kQ_2q}{(D_1-L)^2}$ (4pts)

Then we have, after substitute them to *)

$\left|\frac{Q_1}{D_1^2}\right| = \left|\frac{Q_2}{(D_1-L)^2}\right|$ (note, $Q_1 > 0$, $Q_2 < 0$, $q < 0$)

plug in all the #, we get $\underline{\underline{D_1 = 4m}}$. (4pts)

b) The potential for charge Q_1 & Q_2 at point B is respectively

$U_1 = \frac{kQ_1}{D_2}$, $U_2 = \frac{kQ_2}{L-D_2}$ (4 pts)

Since the total potential energy for q is zero at B, then its potential is also zero.

$U_1 + U_2 = 0$ (4pts)

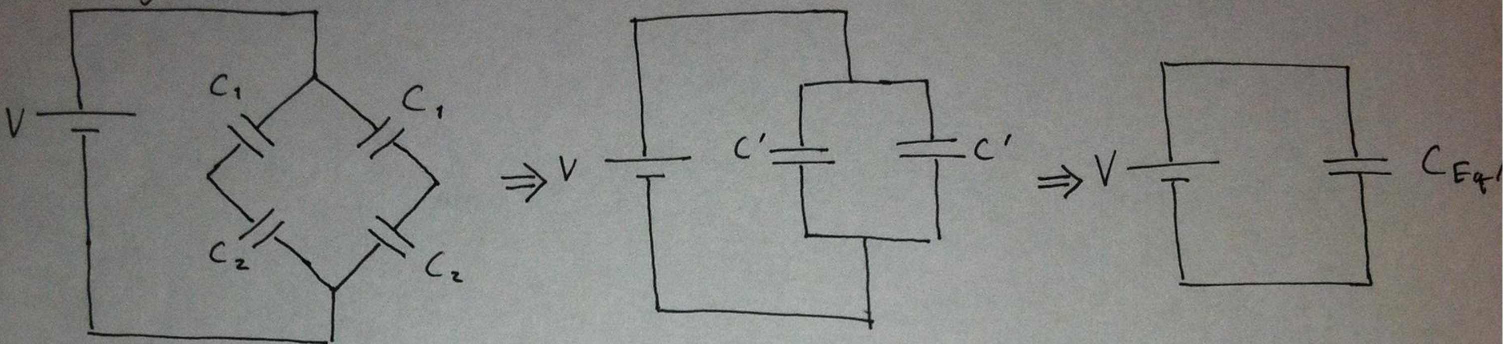
Subst. U_1 & U_2 into it: $\left|\frac{kQ_1}{D_2}\right| = \left|\frac{kQ_2}{L-D_2}\right|$, plug in all #, we have

$\underline{\underline{D_2 = 1.6m}}$

② PART A

We can find C_{EqA} and C_{EqB} in terms of C_1 and C_2 then solve for C_1.

Configuration A:



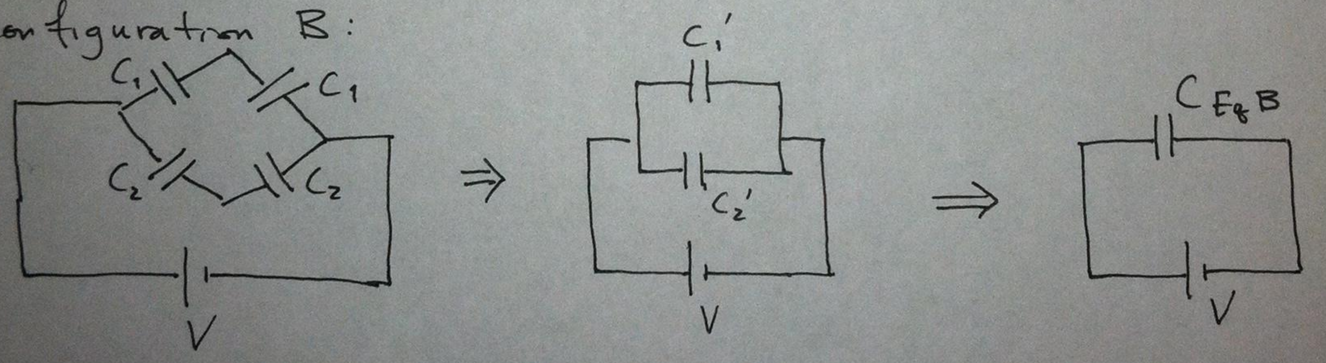
To find C', we must use the fact that C_1 and C_2 are in ~~parallel~~ series in both branches. For capacitors in series, we add their reciprocals.

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \Rightarrow C' = \frac{C_1 C_2}{C_1 + C_2}$$

Then to find C_{EqA}, we use the fact that the C' are in parallel. For capacitors in parallel, we add their capacitance

$$C_{EqA} = C' + C' = 2C' = \frac{2C_1 C_2}{C_1 + C_2}$$

Configuration B:



In configuration B, two C_1 are in series and two C_2 are in series. We add their reciprocals to find C_1' and C_2'.

$$\frac{1}{C_1'} = \frac{1}{C_1} + \frac{1}{C_1} = \frac{2}{C_1} \Rightarrow C_1' = \frac{C_1}{2}$$

$$\frac{1}{C_2'} = \frac{1}{C_2} + \frac{1}{C_2} = \frac{2}{C_2} \Rightarrow C_2' = \frac{C_2}{2}$$

To find C_{EqB}, we use the fact that C_1' and C_2' are in para

$$C_{EqB} = C_1' + C_2' = \frac{C_1}{2} + \frac{C_2}{2} = \frac{C_1 + C_2}{2}$$

We are told that $C_{Eq A} C_{Eq B} = 6F^2$. We use our results above.

$$C_{Eq A} C_{Eq B} = \left(2 \frac{C_1 C_2}{C_1 + C_2}\right)\left(\frac{C_1 + C_2}{2}\right) = C_1 C_2$$

Thus,

$$C_1 C_2 = 6F^2$$

and

$$C_1 = \frac{6F^2}{C_2} = \frac{6F^2}{3F} = 2F.$$

PART B

The energy stored in a capacitor is given by

$$U = \frac{1}{2} C V^2$$

In PART A, we reduced each configuration to $C_{Eq A}$ and $C_{Eq B}$.

$$U_A = \frac{1}{2} C_{Eq A} V^2$$

$$= \frac{1}{2}\left(\frac{2C_1C_2}{C_1+C_2}\right)V^2 = \left(\frac{C_1C_2}{C_1+C_2}\right)V^2 = \left(\frac{6F^2}{3F+2F}\right)(10\,V)^2$$

$$= 120\ J$$

$$U_B = \frac{1}{2} C_{Eq B} V^2$$

$$= \frac{1}{2}\left(\frac{C_1+C_2}{2}\right)V^2 = \left(\frac{C_1+C_2}{4}\right)V^2 = \left(\frac{3F+2F}{4}\right)(10\,V)^2$$

$$= 125\ J$$

Thus, configuration B stores more.

SOLUTION TO PROBLEM 3

We build up the charge distribution given in the problem by the following procedure:

L: a uniformly charged cylinder w/ charge density ρ and radius R

S: a uniformly charged cylinder w/ charge density $-\rho$ and radius $\frac{R}{2}$.

Assemble L & S in the way such that S is placed at the "cylindrical pocket" position, so that the charge distribution we build up is equivalent to ~~that~~ which stated in the problem.

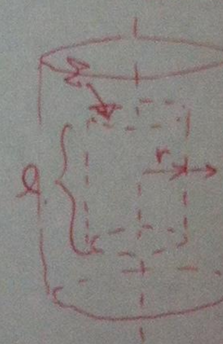
According to the superposition principle, we can calculate the electric fields contributed by L & S separately and sum them up vectorially.

① Point A

We start with deriving the E-field distribution $\boxed{\text{within}}$ a cylinder.

$$\oint_{\Sigma} \vec{E} \cdot d\vec{s} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Rightarrow 2\pi r l E = \frac{\rho \pi r^2 l}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho r}{2\epsilon_0}}$$

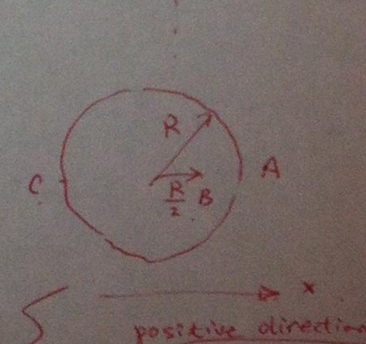


In the following we will apply this formula directly (but for A we will be more illustrative)

$$E_A^{(L)}: \quad 2\pi R E_A^{(L)} = \frac{\rho \pi R^2}{\epsilon_0} \Rightarrow E_A^{(L)} = \frac{\rho R}{2\epsilon_0}$$

$$E_A^{(S)}: \quad 2\pi \left(\frac{R}{2}\right) E_A^{(S)} = \frac{(-\rho)\pi \left(\frac{R}{2}\right)^2}{\epsilon_0} \Rightarrow E_A^{(S)} = -\frac{\rho R}{4\epsilon_0}$$

$$\Rightarrow \boxed{E_A = +\frac{\rho R}{4\epsilon_0}}$$



The electric fields are all lying along the x-axis by symmetry arguments. "+" for positive x direction.

② Point B

$$E_B^{(L)} = \frac{\rho\left(\frac{R}{2}\right)}{2\epsilon_0} = \frac{\rho R}{4\epsilon_0}$$

$$E_B^{(S)} = 0$$

$$\Rightarrow \boxed{E_B = \frac{\rho R}{4\epsilon_0}}$$

③ Point C

$$E_C^{(L)} = -\frac{\rho R}{2\epsilon_0}$$

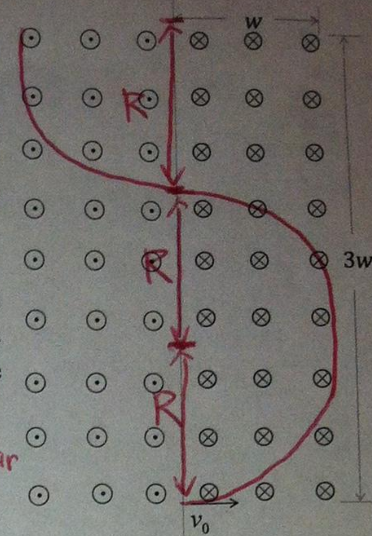
But C is <u>outside</u> of S instead of inside. Apply Gauss's Law separately.

$$\left(-E_C^{(S)}\right) 2\pi\left(R + \frac{R}{2}\right) = \frac{(-\rho)\left(\pi\left(\frac{R}{2}\right)^2\right)}{\epsilon_0} \Rightarrow E_C^{(S)} = \frac{\rho R}{12\epsilon_0}$$

$$\Rightarrow \boxed{E_C = -\frac{5\rho R}{12\epsilon_0}}$$

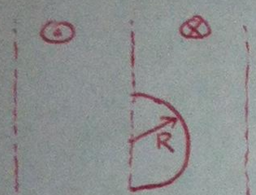
Note that you are supposed to derive the electric field distribution by applying Gauss's Law, instead of copying the result down directly.

4. The figure to the right shows two regions of space. The magnetic field in each region has the same *magnitude*, B, but points in opposite directions in each. Each region has width, w, and length, $3w$. A proton with charge, e, and mass, m, is shown moving with a horizontal velocity just inside the bottom of magnetic field. The proton eventually reaches the top edge of the two regions.



a. If the proton is to stay within the regions while it moves upward, what is the smallest that w can be? Express it in terms of B, v_0, m and e. (I strongly recommend that you sketch the path of the proton through the field.)

A charged particle with velocity perpendicular to $\vec{B}$ will undergo uniform circular motion.



In order to keep the particle inside the $\vec{B}$ field, $w \geq R$. (where R is the radius of the circular motion)

Therefore

$$\boxed{w_{min} = R = \frac{mv_0}{eB}.}$$

b. Take w to be equal to the smallest w that you obtained in part a. How much time, T, does it take the proton to travel to the top edge of the regions? Express it in terms of B, m and e. (If you cannot get part a, you can express your answer in terms of w as well for partial credit.)

As shown in the upper right figure, if $w = R$, then the particle travels a total distance

$$S = \frac{3}{4} \times 2\pi R = \frac{3}{2}\pi R = \frac{3\pi m v_0}{2eB}.$$

$$\boxed{T = \frac{S}{v_0} = \frac{3\pi m}{2eB}}$$

c. What is the velocity (magnitude and direction) of the proton when it leaves the field?

The speed of the particle does not change. ($\because \vec{F} \perp \vec{v}$)

So $|\vec{v}| = v_0$.

As shown in the upper right figure, $\vec{v}$ points upward.