Physics 8B, Section 2 (Speliotopoulos)
Final Examination, Fall 2013
Berkeley, CA

Rules: *This midterm is closed book and closed notes. You are allowed two sides of one and one-half sheets of 8.5" x 11" paper on which you can write whatever notes you wish. You are* ***not*** *allowed to use calculators of any type, and any cellular phones must remain off and in your bags for the duration of the exam.* ***Any violation of these rules constitutes an act of academic dishonesty, and will be treated as such.***

Numerical calculations: *This exam consists of six problems, and each one is worth 20 points. Five of the problems ask you to calculate numbers. I have chosen the parameters in these three problems so that the answers can be expressed in terms of rational numbers. However, if you find that in your calculation of these problems you end up with an expression which you cannot evaluate, simplify the expression as much as you can and leave it.* ***A table of integrals is given on the last page.***

We will give partial credit on this midterm*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any problems, just raise your hand.

Before the exam begins, fill in the following information:

Name: ______________________________ *Disc Sec Number:* ______________________

Signature: ___________________________ *Disc Sec GSI:* _________________________

Student ID Number: __________________ *Disc Sec Time:* _________________________

You must show your student ID when you hand in your exam!

1	
2	
3	
4	
5	
6	
Total	

1. Six electrons (a spin-1/2 particle that obeys the Pauli exclusion principle) are placed inside a *two*-dimensional, infinite square well with side, L. The energy levels of this square well are

$$\frac{E_{n_x n_y}}{E_0} = n_x^2 + n_y^2, \qquad \text{where} \qquad E_0 = \frac{\pi^2 \hbar^2}{2 m_e L^2}$$

and $n_x = 1, 2, 3, \ldots$ and $n_y = 1, 2, 3, \ldots$. Neglect the forces between electrons.

a. The quantum state of an electron is determined by the quantum numbers n_x, n_y and m_s. Determine the quantum states of the six electrons if they are in their lowest energy state, and fill out the following table.

Electron	n_x	n_y	m_s
1			
2			
3			
4			
5			
6			

b. If E_T is the total energy of the six electrons in the well, what is E_T/E_0?

c. A seventh electron is added to the box. If E_7 is the amount of energy required to do so, what is E_7/E_0?

d. Suppose that electrons did not have to obey the Pauli exclusion principle. Going back to part a, what would E_T/E_0 be in this case?

2. A simple harmonic oscillator consists of a particle with mass, m, in a potential well,

$$U(x) = \frac{1}{2}m\omega^2 x^2.$$

The particle is allowed to be anywhere on the real line (so that $-\infty < x < \infty$). The ground state wavefunction for the oscillator is the following:

$$\psi(x) = Ae^{-\left(\frac{m\omega}{2\hbar}\right)x^2},$$

where A is a normalization constant. Remember that there is a table of integrals on the last page. Remember also that

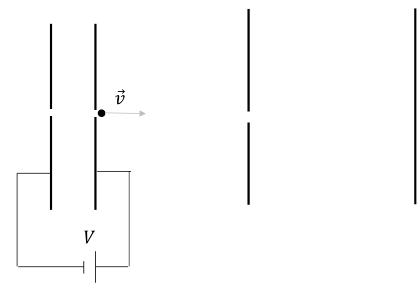
$$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)\psi^2 dx\,.$$

a. What is A?

b. What is the average position of the particle, $\langle x \rangle$?

c. What is the average potential energy of the particle, $\langle U(x) \rangle$?

3. A single slit diffraction experiment was originally done with x-rays that have a wavelength, λ. Thelma wants to do the same experiment, but using electrons (which has mass, m_e) with the same deBroglie wavelength. Her set up is shown to the right. She will accelerate the electrons with a parallel plate capacitor with accelerating voltage, V.



   a. If v is the velocity of electrons after it leaves the capacitor, what must v be if the electrons have the same wavelength as the x-rays? Express it in terms of λ, m_e, and h.

   b. What is the kinetic energy, K, of the electron be? Express it in terms of λ, m_e, and h. (Assume that $v/c \ll 1.$)

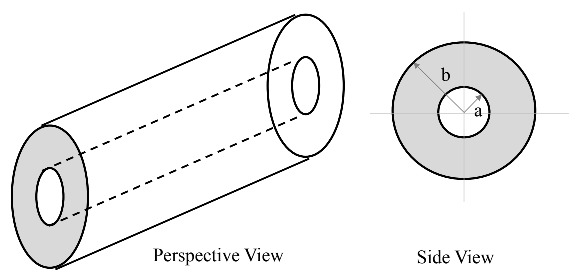
   c. Taking the charge of the electron to be e, what, then, must the accelerating voltage be to produce electrons with this wavelength? Assume that when the electron enters the capacitor its velocity is zero.

4. An observer on Earth sees two spaceships traveling in the same direction. Spaceship A has a velocity, $v_A/c = 4/5$, while spaceship B has a velocity, $v_B/c = 3/5$. At a time $t_0 = 0$, the position of spaceship A is $x_A = 0$ and the position of spaceship B is $x_B = 12\,\mathrm{lmin}$, all measured in the Earth's frame.
   a. Eventually, spaceship A catches up with spaceship B at a time t_f in the Earth's frame. What is t_f? (Note that $\Delta t = t_f - t_0$ is the time it took spaceship A to catch up with spaceship B.)

   b. What is the position, x_A, of spaceship A when it catches up with spaceship B in the Earth's frame? (Note that that $\Delta x = x_f - x_0$ is the distance spaceship A traveled to catch up with spaceship B.)

c. What is the time, $\Delta t'_A$, it took to catch spaceship B, and the distance it traveled, $\Delta x'_A$, in the spaceship A's frame?

5. The figure on the right shows an infinitely long, hollow-core wire (metal is only present in the region between $a \leq r \leq b$). A current, I, flows in the wire, and it is distributed uniformly across the cross-section of the metal between.



a. What is $B(r)$ for $r < a$?

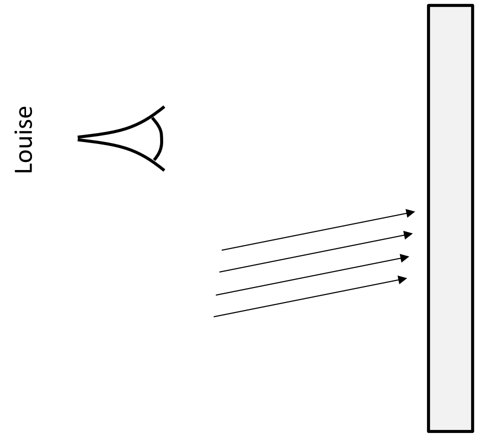
b. What is $B(r)$ for $r > b$?

c. What is $B(r)$ for $a \leq r \leq b$?

6. The figure to the right shows a soap film with index of refraction, $n = 2.5$. White light shines on the film from the left. Louise looks at the film from the left. The thickness, t, of the film decreases with time as

$$t = t_0\left(1 - \frac{T}{T_0}\right),$$

where t is in nanometers, $t_0 = 100\text{ nm}$, $T_0 = 100\text{ s}$, and T is time in seconds. Louise starts looking at the film at $T = 0$.



a. What is the thickness of the film, t_{red}, in nanometers when it looks red? The wavelength of red light is 700 nm.

b. At what time, T_{red}, does it look red?

c. At what **time**, T_{blue}, does it look blue? The wavelength of blue light is 400 nm.

Table of Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax} dx = \frac{(ax-1)e^{ax}}{a^2}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2} dx = 0$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$\int (\sin kx)^2 dx = \frac{1}{2}\left(x - \frac{1}{2k}\sin(2kx)\right)$$

$$\int x(\sin kx)^2 dx = \frac{1}{4}\left(x^2 - \frac{x}{4k}\sin(2kx) - \frac{1}{2k^2}\cos(2kx)\right)$$

$$\int (\mathrm{x}\sin kx)^2 dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{(2(kx)^2-1)}{4k^3}\sin(2kx) - \frac{x}{2k^2}\cos(2kx)\right)$$