105 Midterm

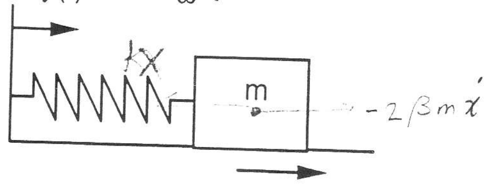
Answer all the questions. You must show the reasoning which leads to your answer to get full credit. Indicate the answers clearly and cross out work you feel is wrong.

1. For a particle of mass m moving as $\bar{r} = 3\bar{i} + t\bar{j} + t^3\bar{k}$, here t is time, find
   (a) the velocity vector $\bar{v}$ and the acceleration vector $\bar{a}$. (2 pts)
   (b) the angular momentum vector $\bar{l}$ (3 pts)
   (c) the force acting on the particle $\bar{F}$. (1 pts)
   (d) the torque acting on the particle $\bar{\tau}$. (2 pts)
   (e) show that $\frac{d\bar{l}}{dt} = \bar{\tau}$. (2pts)

2. A mass m drops from rest in air. If the air friction is proportional to the square of the velocity, i.e., $f=-bv^2$,
   (a) what is the terminal speed v_t? (2 pts).
   (b) Find the speed as a function of time? (4 pts) Hint: $\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$.
   (c) What is the answer (b) in the limit of $t \to \infty$? (2 pts).
   (d) What would you expect of the answer (b) in the limit of $t \ll v_t/g$? (2 pts).

3. A mass of m is moving upward from the surface of the earth with an initial velocity v_0. The mass and radius of the earth are M and R, respectively. Assuming that the v_0 is big enough so that you can not assume a constant gravitational field.
   (a) How high above the earth can the mass eventually reach? (5 pts)
   (b) For $v_0 \ll \sqrt{\frac{GM}{R}}$, how would the result (a) approach to? Express your result in term of the gravitational field at the surface of the earth. (5 pts)

4. A mass m is on a frictionless table and is attached to the wall by a massless spring (spring constant k). During an earthquake, the wall moves left and right as $y(t) = B\cos\omega t$. The air friction on the block is $f=-2\beta mv$.



   (a) Show that the eqn. of motion for m is $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \omega_0^2 B\cos\omega t$ with $\omega_0 = \sqrt{\frac{k}{m}}$ (3 pts)
   (b) Derive the amplitude of the steady-state motion. (5 pts)
   (c) What is the amplitude at the resonance frequency $\omega = \omega_0$? (2 pts)