PLEASE DO NOT OPEN THIS COVER SHEET UNTIL TOLD TO DO SO

PRINT YOUR NAME	SIGN YOUR NAME

University of California, Berkeley
Physics 105, Section 1 - Spring Semester 1994

SECOND MIDTERM EXAMINATION
Friday, March 25

INSTRUCTIONS;

1. This is an open-book examination; but you may use only (1) our textbook, (2) your class notes, (3) your problems and the distributed solutions, (4) integral tables, and (5) a calculator.

2. Write ONLY on the pages of this exam. Put each answer in its $\boxed{\text{box}}$ so it can be found easily when scoring your work. In the space by each question write your steps to the answer; be brief, but clear. Write equations, not sentences, unless required. YOU MUST SHOW YOUR STEPS ! No credit will be given for unsupported results, or for illegible work. Do scratch work entirely on the exam; if more space is needed use the backs of the sheets. Do not separate the sheets.

3. The points for each part are shown in the left margin; the total is 100 points.

4. KEEP MOVING! Do not get bogged down. Before starting, scan the entire exam, marking parts you think you can do well; then do them first. If you get stuck, or are moving slowly, skip that part and move on, returning if time remains. A COMMON REASON FOR A POOR SCORE IS FAILING TO WORK AT ALL ON AN EASY PART THAT COMES LATE IN THE EXAM.

PROBLEM	POINTS
1	
2	
3	
TOTAL	

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Second Midterm Exam - page 1

PROBLEM 1

A particular Lagrangian function for a particle with charge q and mass m is:

$$L = (m/2)(\dot{x}^2 + \dot{y}^2 + \dot{z}^2] + q B \dot{z} x .$$

B is a constant parameter; it is a magnetic field strength.

(6) (a) Evaluate the canonical momenta p_x, p_y, and p_z.

$p_x =$
$p_y =$
$p_z =$

(6) (b) Find Lagrange's equations of motion for x and z .

(6) (c) Find expressions for three constants of the motion (first integrals); these are functions of x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$, and the constants m, q, and B.

PROBLEM 1 (continued)

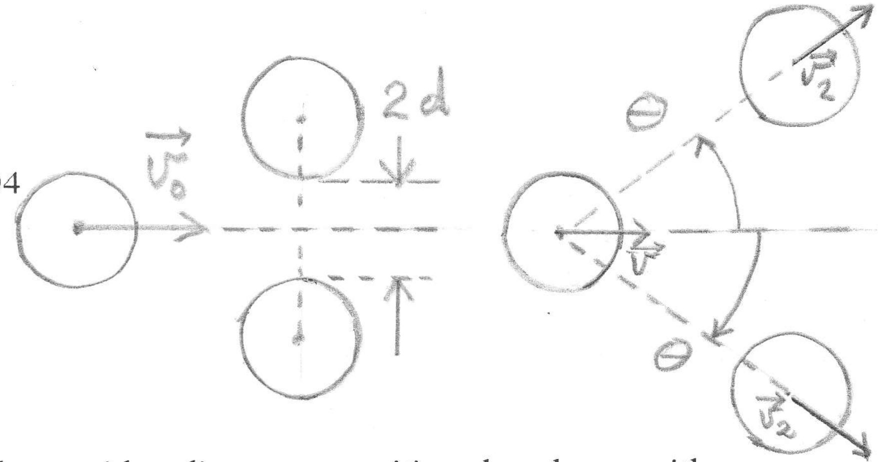
(6) (d) Find the Hamiltonian H as a function of x, y, z, p_x, p_y, p_z and the constants.

H =

(6) (e) Find $x(t)$ and $z(t)$.

x(t) =
z(t) =

(4) (f) Write Hamilton's canonical equations of motion for x and p_x.



PROBLEM 2

Three identical smooth hard spheres with radius a are positioned as shown, with two at rest separated by a distance $2d = 2a/5$ and the third moving toward them with velocity $\mathbf{v_0}$ along a line perpendicular to and bisecting their line of centers. After they collide the first two move away with speed v_2 at equal angles θ from $\mathbf{v_0}$ and the third has velocity $\mathbf{v}$ parallel to $\mathbf{v_0}$.

(10) (a) Write the equations for conservation of momentum and of energy.

(8) (b) Find $\sin\theta$ and $\cos\theta$.

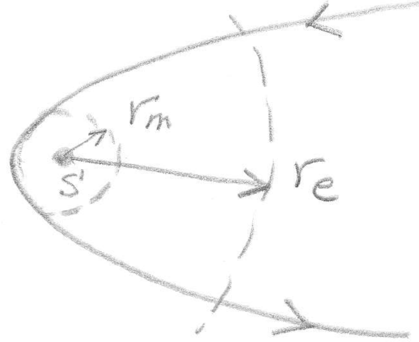
$\sin\theta =$
$\cos\theta =$

(8) (c) Find v_2/v_0.

$\boxed{v_2/v_0 =\quad}$

(8) (d) Find v/v_0.

$\boxed{v/v_0 =\quad}$



PROBLEM 3

For long-period comets (> 200 yr) the aphelion is very far from the sun; because the major axis $2a$ is so large, the total energy $E = - k/2a$ is very small, and can be put to zero in this problem. Consider such a comet whose perihelion is at the orbit of Mercury, a distance $r_m' = 0.387$ AU from the sun. How much time T does it spend inside the earth's orbit ($r_e' = 1$ AU) during one pass around the sun? Give your final answer in years , ands show that this number depends only on pure numbers such as π and $2^{1/2}$ and the dimensionless ratio of radii 0.387 AU/1 AU = 0.387 .

PROCEDURE:

(6) (a) Prove that $(L/m)^2 = 2GMr_m$ with L = angular momentum , M = solar mass, and m = comet mass. (This only takes 2 or 3 lines; if you can't do it, use the result below.)

(8) (b) Use the energy equation $E = (m/2)[\dot{r}^2 + (L/mr)^2] - GMm/r \approx 0$ (text, p. 248), and the result from (a) above, to find $\dot{r} = dr/dt$ in terms of $2GM$, r, and r_m only .

$\boxed{dr/dt =}$

PROBLEM 3 (continued)

(10) (c) Separate the variables r and t, and integrate on r from r_m to r_e to evaluate T/2. Write T in the form

$$T = (r_e^3/(2GM))^{1/2} f(x),$$

with $x = r_m/r_e$; that is, find the function f(x). [Write the physical constants out front so that what remains, your f(x), is dimensionless.]

You may confront one of these integrals: $\int (y-1)^{-1/2}\,dy = 2(y-1)^{1/2}$;
$\int y\,(y-1)^{-1/2}\,dy = (2/3)(y+2)(y-1)^{1/2}$; $\int y^{-1}(y-1)^{-1/2}\,dy = 2\tan^{-1}(y-1)^{1/2}$.
[HINT: The y in these integrals is r/r_m, NOT x.]

f(x) =

(6) (d) [An independent question] Express $r_e^3/(2GM)$ in terms of τ_e = 1 yr. (text, p. 259)

$\frac{r_e^3}{2GM} =$

(2) (e) Using the results of (c) and (d), find T in years.

T = YR.