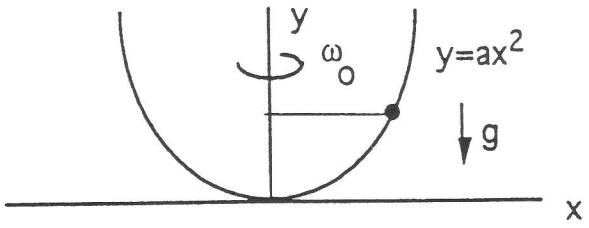
(2)

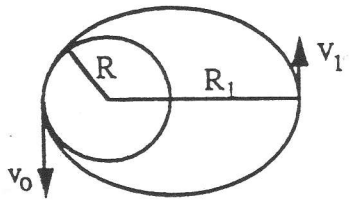
105 Midterm-2 (Fall, 1997)

Answer all the questions. You must show the reasoning which leads to your answer to get full credit. Indicate the answers clearly and cross out work you feel is wrong.

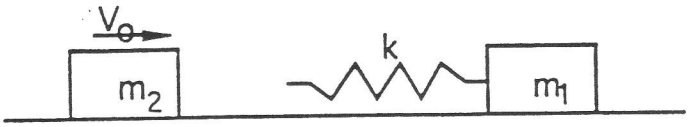
10 1. A wire of the shape of $y=ax^2$ is rotating around its vertical axis with an angular velocity ω_0, a point mass m is moving frictionlessly on the wire under the gravitational force.
(a) Write down the Lagrangian in terms of x and $\dot{x}$. (5 pts)
(b) Derive the equation of motion for x. (5 pts)



15 2. A satellite of mass m (m<<M) is launched from the Earth horizontally with a speed of v_0 into an elliptic orbit (see figure below).
(a) What are the energy and the angular momentum of the satellite? (5 pts)
(b) What is the farthest distance R_1 that the satellite can reach? (5pts)
(c) What is the speed v_1 of the satellite at R_1? (5 pts)
Write down the results in terms of G, M, m, R, and v_0. Here G, M, and R are the gravitational constant, the mass of the Earth, and the radius of the Earth, respectively.



20 3. A massless spring (spring constant k) is attached to a block of mass m_1 which is at rest on a frictionless table. Another block of mass m_2, moving from the left with a velocity v_0, collides elastically with the first block.
(a) What is velocity of the center of mass (CM), V_c? (2 pts)
(b) What are the velocities of m_1 and m_2 in the CM frame? (3 pts)
(c) What's the total mechanical energy of this system in the CM frame? (5 pts)
(d) Show that the total linear momentum in the CM frame is zero. (5 pts)
(e) What's the maximum compression of the spring during the collision? (5 pts)



10 4. A particle is projected horizontally towards the east at a height of h above the surface of the Earth at a northern latitude λ with a velocity of magnitude v_0, show that the lateral deflection when the particle strikes the earth is $d=(2hv_0\omega\sin\lambda)/g$. Here ω is the spinning angular velocity of the earth. (10 pts)