(9)

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University of California, Berkeley
Physics 105, Section 1 - Fall 1991

FINAL EXAMINATION

INSTRUCTIONS:

1. This is a CLOSED BOOK examination. You may use a calculator.
2. Write ONLY on the pages of this exam. Put each answer in its box so it can easily be found when scoring your work. In the space below or near each question write your steps to the answer; be brief, but clear. Write equations, not sentences, unless required.

YOU MUST SHOW YOUR STEPS!

No credit will be given for unsupported results, or for illegible work. Do scratch work entirely on the exam. If more space is needed use the backs of the sheets. Do not separate the sheets.

3. The points for each part are shown in the left margin; the total is 100 points.

PROBLEM	POINTS
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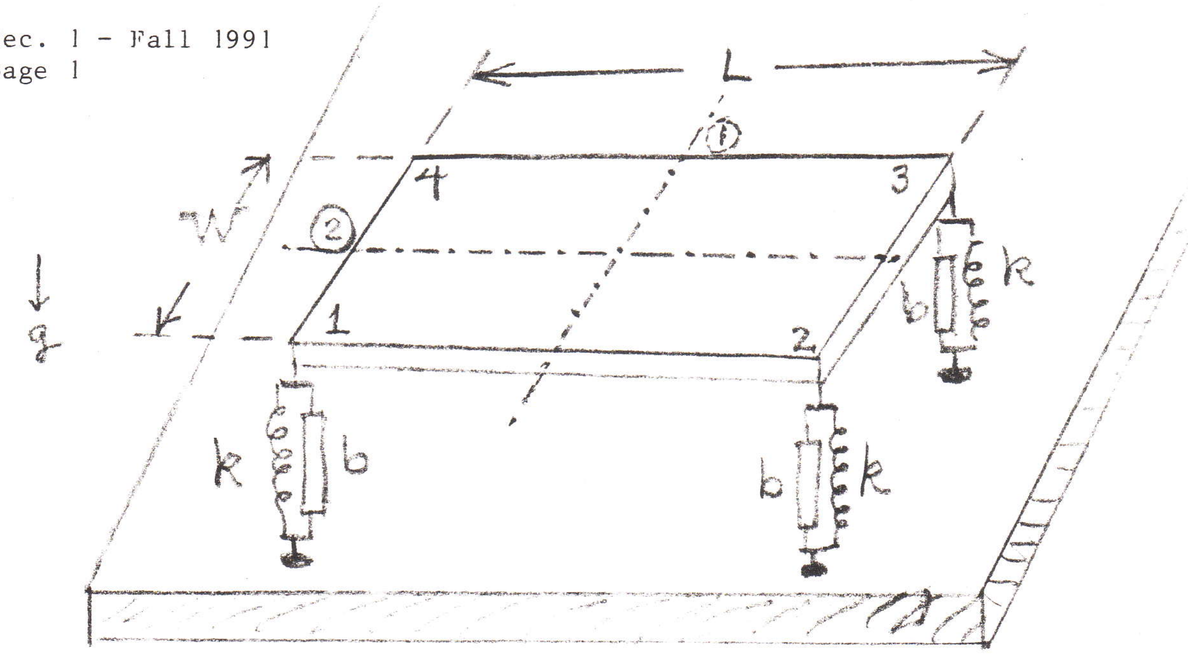
8	
TOTAL	

PROBLEM 1



An automobile and its suspension system are idealized as a thin uniform rectangular plate of length L, width W, and mass M, supported by four identical springs (each having force constant k) at its corners. At each spring one may mount a shock absorber (damper) producing a force $-b\,dx/dt$ if its length x is changing. The vertical coordinate x_i measures the height of the ith corner (i = 1,2,3,4) above its equilibrium position in which the plate is horizontal. The shock absorbers are selected so that the uniform vertical oscillation of the plate ($x_1 = x_2 = x_3 = x_4$) is critically damped when they are mounted.

(4) (a) Find the frequency ω_0 of this vertical oscillation if the shock absorbers are absent.

$\omega_0 =$

(4) (b) Find the value of the damping parameter b required to critically damp this motion.

b =

PROBLEM 1 (continued)

c) There are two other normal modes of oscillation of this system: Pitch, which is a rotational oscillation about axis ① ($x_1 = x_4 = -x_2 = -x_3$), and Roll, which is a rotational oscillation about axis ② ($x_1 = x_2 = -x_3 = -x_4$). Find the frequencies ω_1 and ω_2 of these modes if the shock absorbers are absent.

$\omega_1 =$
$\omega_2 =$

(3) (d) If the four shock absorbers are mounted, are these modes underdamped, critically damped, or overdamped? Give a simple calculation to justify your answer. HINT: Calculate $\omega^2 = \omega_o^2 - \beta^2$, and remember that for a simple oscillator $\beta = b/2m$.

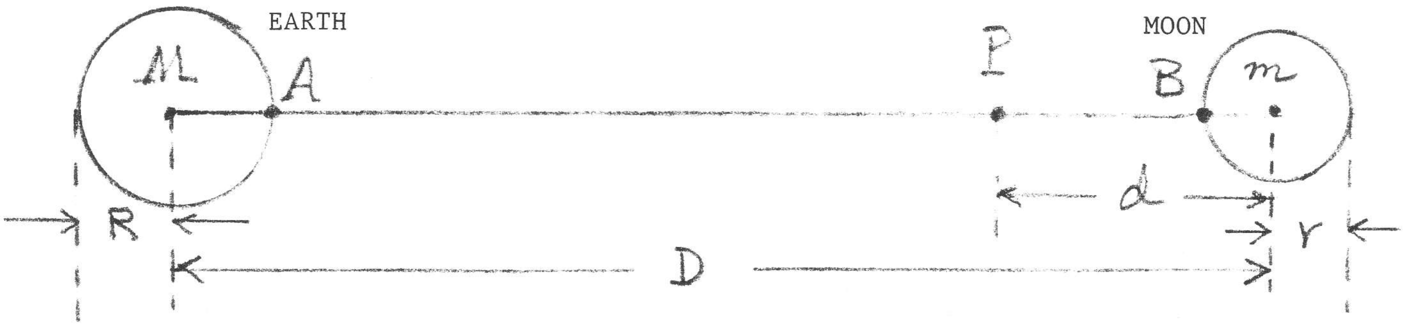
$\rightarrow = \omega_1^2$ OR ω_2^2 IN (c)

CHECK ONE

- ☐ underdamped
- ☐ critically damped
- ☐ overdamped

PROBLEM 2

Imagine the earth and the moon to be at rest and not rotating. At a point P along the line between their centers their gravitational fields cancel.



Dealing only with dimensionless ratios, determine the distance d of P from the moon's center in terms of D (see sketch). The dimensionless ratios to be used are:

$$d/D = \rho, \quad M/m = \mu .$$

The actual value of μ is almost exactly 81, so assume $\mu = 81$ exactly. Find and solve a quadratic equation for ρ in which μ appears as a parameter.

(5) (a)

put quadratic equation in its simplest form here, containing ρ and μ.

(4) (b)

solution: ρ =

put a number here, using $\mu = 81$.

PROBLEM 2 (Continued)

c) Imagine that a space ship leaves the moon's surface at point B with just barely more velocity than needed to reach point P by coasting. If it continues coasting along the line it will arrive at point A on earth with velocity v_A. Write (but do not try to solve) an equation whose solution would give the value of v_A. This equation may contain as many of the following quantities as you need:

d, D, R, r, M, m, G, g, and (of course) v_A.

Draw a box around your equation.

PROBLEM 3

A magnetron is an oscillator which generates high-frequency electromagnetic waves suitable for use in radar and in microwave cooking ovens. In it electrons move in planes z = constant between two cylinders r = a and r = b (cylindrical polar coordinates). There is a uniform magnetic field $B_o\hat{e}_z$ parallel to the z axis, and a radial electric field $E(r)\hat{e}_r$ between the cylinders, which has the electric potential

$$V(r) = -\int^r E(r')\, dr' .$$

For an electron of mass m and electric charge q, the kinetic and potential energies are:

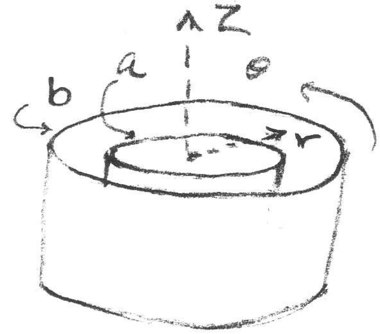
$$T = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) , \quad U = qV(r) .$$

The Lagrangian function is $L = T - U + q\,\vec{A}\cdot\vec{v}$. For the uniform field, $\vec{A} = \tfrac{1}{2}B_o r\hat{e}_\theta$ so that $q\,\vec{A}\cdot\vec{v} = \tfrac{1}{2}qB_o r^2\dot{\theta}$.

(6) (a) From this Lagrangian calculate the Lagrangian equations of motion for $\ddot{r}$ and $\ddot{\theta}$. [They will contain the unspecified quantity E(r)]

equation with $\ddot{r}$

equation with $\ddot{\theta}$

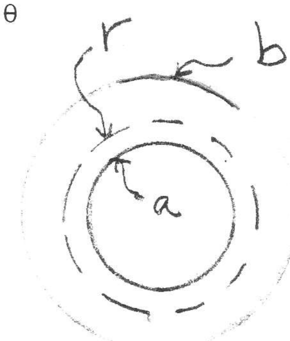


PROBLEM 3 (Continued)

b) Consider the conditions required to allow an electron to move steadily in a circle of radius r between a and b. Show that if B_o, $E(r)$, q, m, and r are regarded as given then there are two different values of the velocity on such a circle unless the value of $E(r)$ is too big and of the wrong sign.

To do this, write a quadratic equation for v_θ for this circular motion which contains the parameters just listed: B_o, $E(r)$, q, m, and r.

quadratic equation for v_θ



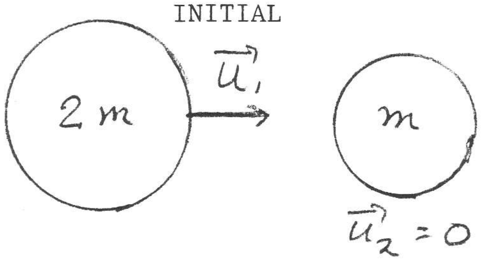
(3) (c) Can you give a simple answer to the question: How can there be TWO DIFFERENT motions along the same circle with different velocities?

PROBLEM 4

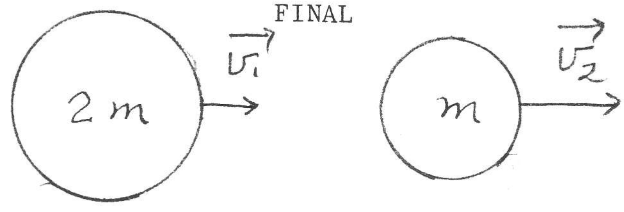
A ball of mass 2m and initial velocity $\vec{u}_1$ collides with a second ball of mass m at rest ($\vec{u}_2 = 0$). After the collision they continue to move along the line of $\vec{u}_1$ with velocities $\vec{v}_1$ and $\vec{v}_2$, respectively (see sketch). The collision is somewhat inelastic, having a coefficient of restitution $e = \frac{1}{2}$. It is defined as the ratio of final relative speed to initial relative speed;

$$e = \left|(v_2 - v_1)/(u_2 - u_1)\right| .$$

INITIAL



FINAL



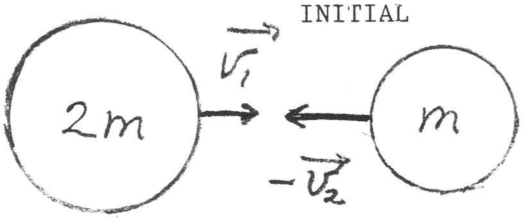
(6) (a) Calculate the ratios v_1/u_1 and v_2/u_1.

$v_1/u_1 =$
$v_2/u_1 =$

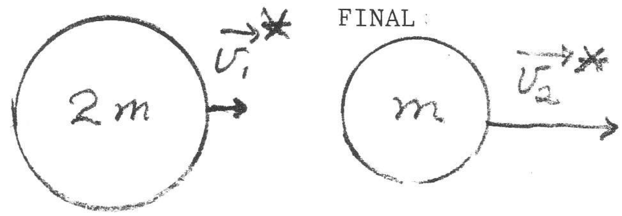
<u>PROBLEM 4</u> (Continued)

(b) After this collision the second ball continues, colliding elastically with a wall and returning with reversed velocity to collide again with the first ball. The initial and final conditions are shown in the sketches below. Calculate the ratios v_1^*/u_1 and v_2^*/u_1.

INITIAL



FINAL



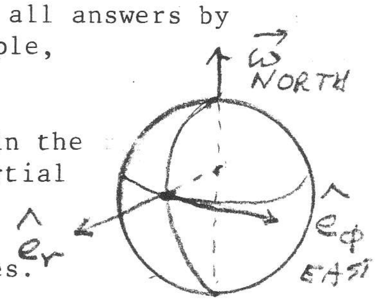
v_1^*/u_1 =
v_2^*/u_1 =

PROBLEM 5

In this problem consider the real and fictitious forces acting on a mass m located on the surface of an ideal smooth spherical earth of radius R, which rotates with constant angular velocity $\vec{\omega} = \omega \hat{e}_z$. The basic relation is

$$\vec{a}_{rot} = \vec{F}/m - 2\vec{\omega} \times \vec{v}_{rot} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

in which $\vec{F} = m\vec{a}_{inertial}$ is the sum of all real forces acting on m. Coriolis and centrifugal accelerations are in the earth-fixed (rotating) coordinates; real forces are, of course, independent of the choice of coordinates. Express all answers by giving magnitude and direction (shown by a unit vector): for example, $\vec{F}_{grav} = -mg\,\hat{e}_r$.



(4) (a) In part (a) the mass is moving along the equator toward the west in the rotating coordinates with speed ωR, so that it is at rest in inertial coordinates; $\vec{v}_{rot} = -\omega R\,\hat{e}_\phi$, $\vec{v}_{inertial} = 0$.

(i) Find the Coriolis acceleration in earth-fixed coordinates.

$\vec{a}_{Coriol}$ =	
magnitude	direction

(ii) Find the centrifugal acceleration in these coordinates.

$\vec{a}_{centrif}$ =	
magnitude	direction

(iii) Find $\vec{F}/m$ and explain your result, including mention of gravity and all other forces.

$\vec{F}/m$ =	
magnitude	direction

PROBLEM 5 (Continued)

(b) In part (b) the mass is at rest on the equator in the rotating system.

(i) Find the Coriolis acceleration.

$\vec{a}_{Coriol}$ =

magnitude direction

(ii) Find the centrifugal acceleration.

$\vec{a}_{centrif}$ =

magnitude direction

(iii) Find $\vec{F}/m$ and explain your result, as in part (a) above.

$\vec{F}/m$ =

magnitude direction

(4) (c) In part (c) the mass moves along the equator with velocity $\vec{v}_{rot} = -\frac{1}{2}\omega R\, \vec{e}_\phi$, the average of the values in (a) and (b) above.

(i) Find the Coriolis acceleration.

$\vec{a}_{Coriol}$ =

magnitude direction

(ii) Find the centrifugal acceleration.

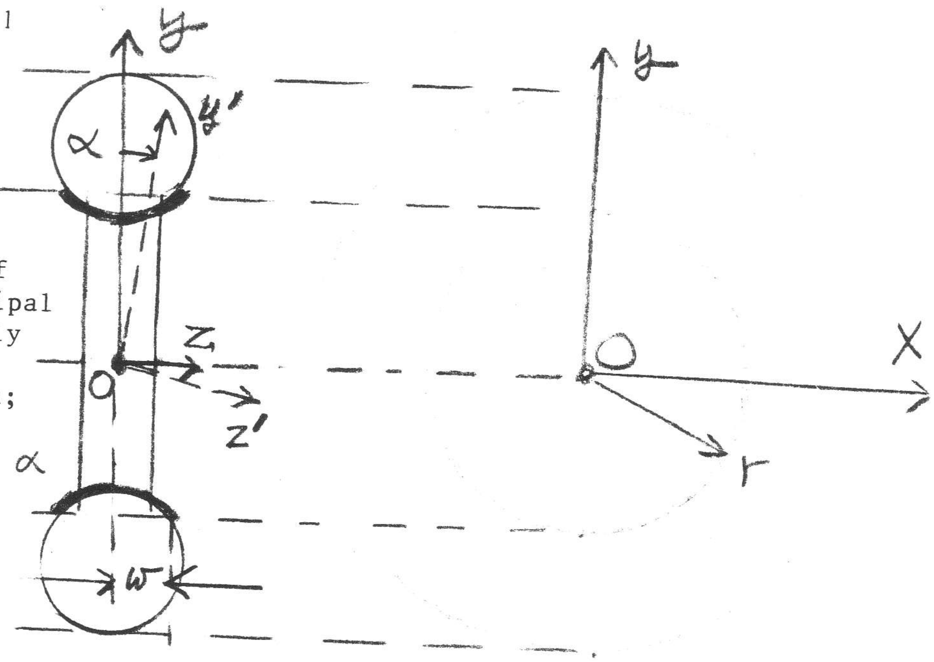
$\vec{a}_{centrif}$ =

magnitude direction

(iii) Find $\vec{F}/m$ and explain your result, again including gravity and any other forces.

$\vec{F}/m$ =

magnitude direction



PROBLEM 6

The center of mass of a car's wheel with tire is found to be at its geometrical center O (see sketch), but its tensor of inertia is found to have principal axes x',y',z' which are slightly tilted from the "symmetrical" body-fixed axes x,y,z, as shown; $x' = x$, $y' = y\cos\alpha + z\sin\alpha$, $z' = -y\sin\alpha + z\cos\alpha$, with α a very small angle (exaggerated in the sketch). When expressed in coordinates x,y,z the tensor of inertia is:

$$\begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & \Delta I \\ 0 & \Delta I & I_3 \end{pmatrix}$$

in which the small products of inertia $I_{23} = I_{32} = \Delta I$ are the result of this defect, which is called dynamic imbalance. If the wheel is rotated steadily about its axle (the z axis) with angular velocity ω_z, the angular momentum will have a component perpendicular to z, because

$$L_i = \sum_j I_{ij}\,\omega_j .$$

(5) (a) Evaluate this component of $\vec{L}$, giving both magnitude and direction (using unit vectors $\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$ in the xyz coordinates).

comp. of L perp. to z =	
magnitude	direction

4) (b) Calculate the torque $\vec{N}$ on the wheel (supplied by its bearings) required to maintain this steady rotation, giving its magnitude and direction (using the unit vectors as in part (a) above).

$\vec{N}$ =	
magnitude	direction

PROBLEM 6 (Continued)

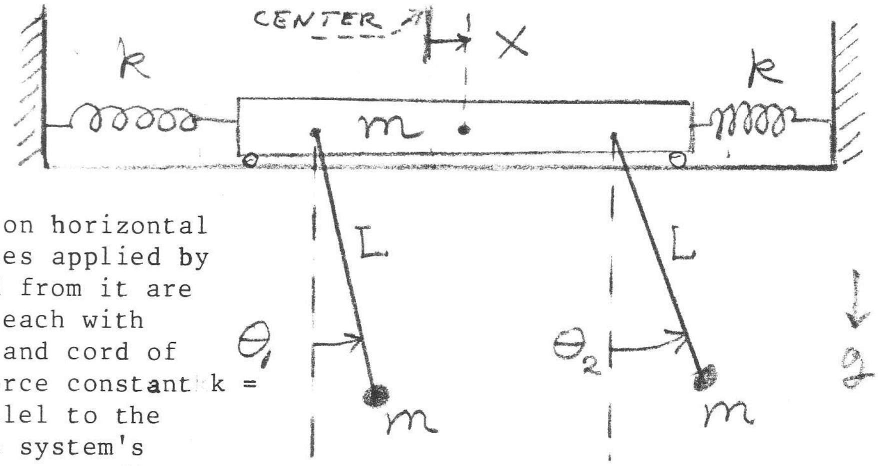
c) By fixing two small masses at appropriate positions in the y-z plane the principal axes may be restored to the desired (xyz) directions without moving the center of mass. (This is called "dynamically balancing the wheel".) The only available positions are along the rims, where $x^2 + y^2 = r^2$ and $z = \pm w$ (see sketch).

Find the coordinates for the positions of these two equal masses, and calculate the amount of mass (at each position) in terms of w, r, and ΔI.

HINT: The tensor of inertia for a single mass is:

$$\begin{pmatrix} m(y^2+z^2) & -mxy & -mxz \\ -mxy & m(x^2+z^2) & -myz \\ -mxz & -myz & m(x^2+y^2) \end{pmatrix}$$

first mass: x =	, y =	, z =	.
second mass: x =	, y =	, z =	.
mass m =			



PROBLEM 7

A cart of mass m rolls smoothly on horizontal rails, subject to centering forces applied by two identical springs. Suspended from it are two identical simple pendulums, each with point mass m (same as the cart) and cord of length L. Each spring has the force constant $k = \frac{1}{2}(3mg)/L$. All motions are parallel to the plane of the sketch, so that the system's position is specified by the three coordinates θ_1, θ_2, and x, measured from equilibrium as indicated.

(6) (a) Without making any calculations at all, give short descriptions in words of the motions in each of the three normal modes. Give all parts of the descriptions which can be known without making any calculations.

(i) MODE 1

(ii) MODE 2

(iii) MODE 3

(3) (b) For small displacements from equilibrium, the kinetic and potential energies are:

$T = \frac{1}{2}m[\dot{x}^2 + (\dot{x} + L\dot{\theta}_1)^2 + (\dot{x} + L\dot{\theta}_2)^2]$, $U = \frac{1}{2}(3mg/L)x^2 + \frac{1}{2}mgL(\theta_1^2 + \theta_2^2)$.

Find the three Lagrange equations of motion.

PROBLEM 7 (Continued)

(c) The secular determinant to be solved for the frequencies of the normal modes is:

$$\begin{vmatrix} 3(\omega_p^2 - \omega^2) & -\omega^2 & -\omega^2 \\ -\omega^2 & \omega_p^2 - \omega^2 & 0 \\ -\omega^2 & 0 & \omega_p^2 - \omega^2 \end{vmatrix} = 0,$$

in which ω_p^2 is an abbreviation for g/L. Find the frequencies of the three normal modes, as fractions of ω_p.

$(\omega_1/\omega_p)^2 =$
$(\omega_2/\omega_p)^2 =$
$(\omega_3/\omega_p)^2 =$

(2) (d) Using your solutions for the normal mode frequencies from part (c), verify and extend your statements in part (a) so as to give a complete description of the motion in each of the three normal modes.

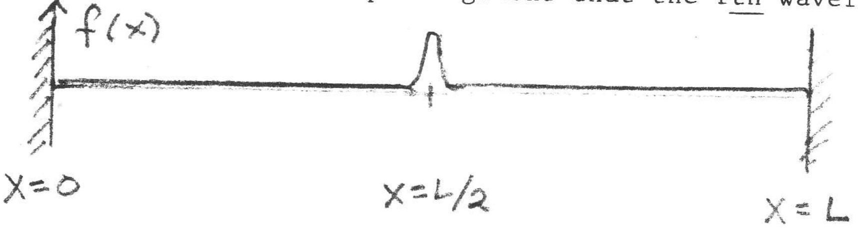
PROBLEM 8

A flexible elastic string is stretched by tension $\mathcal{T}$ to length L between two supports; its mass per unit length M/L is ρ. Transverse waves of any shape may travel along the string with the same velocity v.

(2) (a) Write the expression for v in terms of the parameters ρ and $\mathcal{T}$. If you do not remember it you may obtain it easily by dimensional analysis.

v =

(2) (b) The string is held in a transversely displaced shape sketched below (with transverse scale greatly exaggerated) and released from rest at t = 0, so that $q(x,0) = f(x)$ and $\dot{q}(x,0) = 0$. Following the release of the string its displacement will contain sinusoidal waves of many wavelengths. Give a simple argument that the rth wavelength λ_r is equal to 2L/r.



(3) (c) Define the class of values of the integer r for which NO waves will be present.

(1) (d) Sketch the shape of the displacement at time $t_d = \frac{1}{4}L(\rho/\mathcal{T})^{\frac{1}{2}}$.

(1) (e) Sketch the shape of the displacement at time $t_e = 3t_d$.

(1) (f) Sketch the shape of the displacement at time $t_f = 4t_d$.

(1) (g) Sketch the shape of the displacement at time $t_g = 7t_d$.

HINT: At time $t = 8t_d$ the shape is as drawn at t = 0.

END OF EXAMINATION