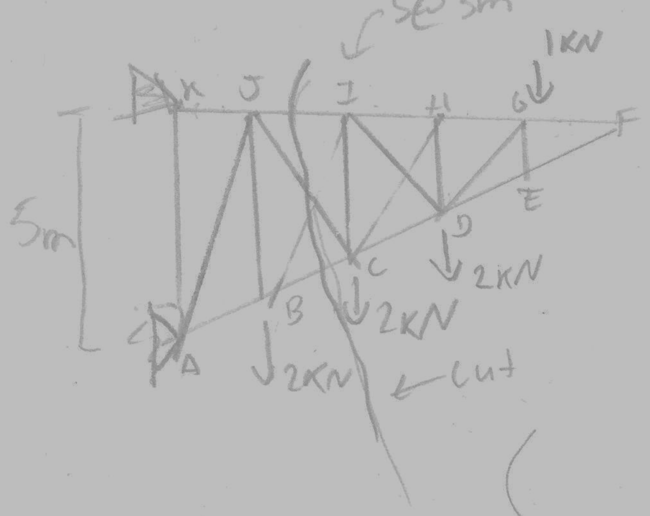
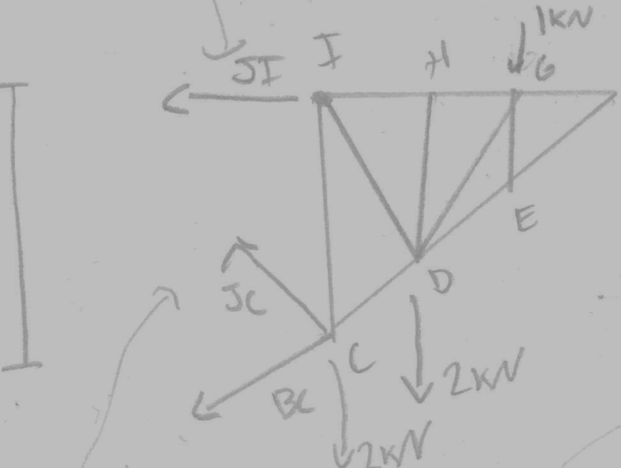
Problem 1



Find internal forces for JI and JC.

I make a cut through JI and JC so I can solve in one go.
It does not matter what side I choose because of equilibrium, but I choose the right because I do not know the reactions @ K and A.

$5\left(\frac{3}{5}\right)=3$



Three unknowns: JI, JC, BC

Three equations!

$\Sigma M_C = 0$

$JI(3) - 2(3) - 1(6) = 0$

$JI = 4 \text{ kN } T$

I assume in tension. If neg. then we know its in compression

$\Sigma F_x = 0$

$-JC\cos\theta - BC\cos\phi - JI = 0$

$JC\cos\theta + BC(\cos\phi) + JI = 0$

$BC = \frac{1}{\cos\phi}(-JI - JC\cos\theta)$

$\theta = 45°$

I didn't calculate but $\tan\phi = \frac{5}{15} = \frac{1}{3}$

$\Sigma F_y = 0$

$JC\sin\theta - BC\sin\phi - 2 - 2 - 1 = 0$

$JC\sin\theta + \tan\phi(JI + JC\cos\theta) = 5$

$JC\sin\theta + \frac{1}{3}(JI + JC\cos\theta) = 5$

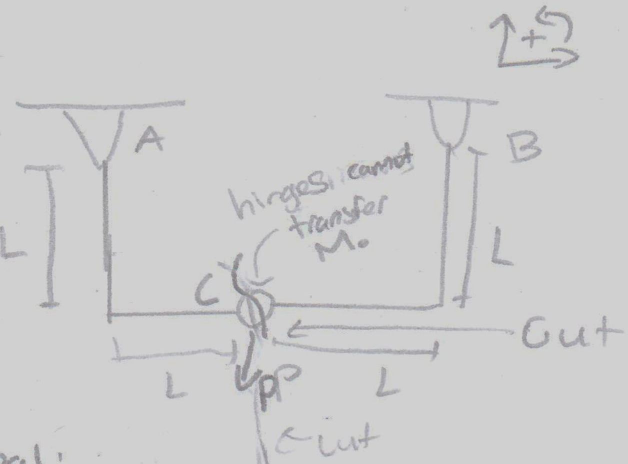
$JC(\sin\theta + \frac{1}{3}\cos\theta) = 5 - \frac{1}{3}JI$

$JC = \frac{5 - \frac{1}{3}JI}{\sin\theta + \frac{1}{3}\cos\theta} = \frac{5 - \frac{1}{3}(4)}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(\frac{1}{3}\right)}$

$JC = 3.9 \text{ kN } T$

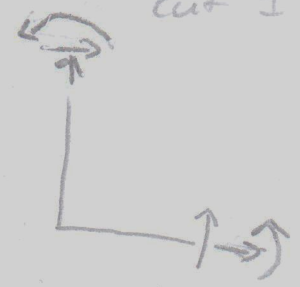
$\boxed{\begin{array}{l} JI = 4 \text{ kN } T \\ JC = 3.9 \text{ kN } T \end{array}}$

Problem 2

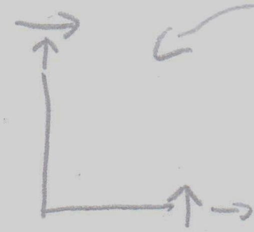


For frames, we solve by making cuts. I make a cut at the Hinge to solve the problem.

In general when I make cut I have the following:

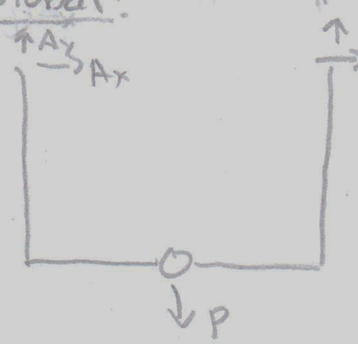


However since there is a hinge @ C and a pin at A, I have this.

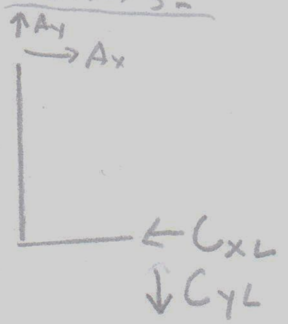


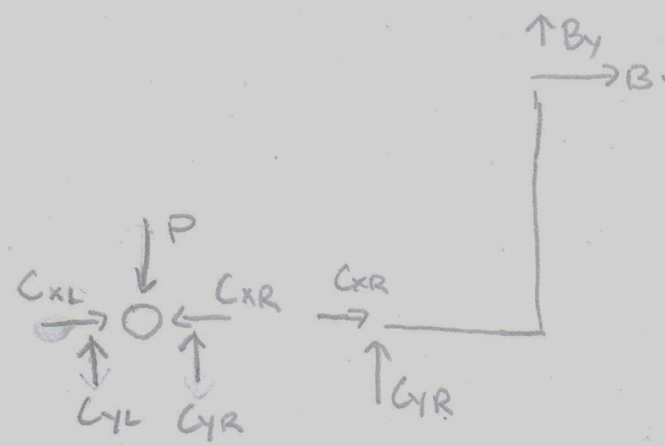
There is no moment @ C or A b/c pins and hinges don't transfer moment.

Global:



Sections:





In the global FBD I have 4 unknowns and 3 equations. However, I have a hinge so I can solve the equations. Let's see why. When I count all of the unknowns in my section I get: $A_y, A_x, C_{xL}, C_{yL}, C_{xR}, C_{yR}, B_y, B_x$, or 8 unknowns. From the sections on the left and right I get 3 equations per sections, so 6 in total. From the point C, I get 2 equations (there is no moment equation). In total I have 8 equations and 8 unknowns so I can solve, or the problem is statically determinant. If I did not have the hinge, I would have the moments C_{ML} and C_{MR} and only one more equation at the point C; the moment equation. This would mean I have 10 unknowns and 9 equations. ⟶

In short, the addition of thie hinge makes the problem statically determinant.

Let's solve:

Global:

$\Sigma F_x = 0$

$A_x = -B_x$

$\Sigma M_A = 0$

$B_y(2L) - P(L)$

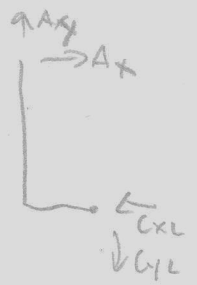
$B_y = P/2$

$\Sigma F_y = 0$

$A_y + B_y = P$

$A_y = P/2$

Left side:



$\Sigma M_C = 0$

$A_x(L) + A_y(L) = 0$

$A_x = -\frac{P}{2}$

Back to global:

$A_x = -B_x$

$B_x = \frac{P}{2}$

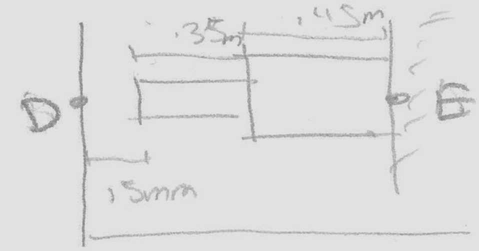
$A_x = -\frac{P}{2}$

$A_y = P/2$

$B_x = -P/2$

$A_y = P/2$

Problem 3:



Bronze:

$A = 1500mm^2 = .0015m^2$

$E = 105\ GPa = 105\times10^9\ Pa$

$\alpha = 21.6\times10^{-6}/°C$

Aluminum

$A = 1800\ mm^2 = .0018\ m^2$

$E = 73\ GPa = 73\times10^9\ Pa$

$\alpha = 23.2\times10^{-6}/°C$

a) Find flexibility constants

$\frac{L}{EA}P = fP$

flexibility constant

Bronze:

$f_B = \frac{L_B}{E_B A_B} = \frac{.35}{(.0015)(105\times10^9)}$

$\boxed{f_B = 2.22\times10^{-9}\ m/N}$

Aluminum:

$\boxed{f_A = \frac{L_A}{E_A A_A} = 3.42\times10^{-9}\ m/N}$

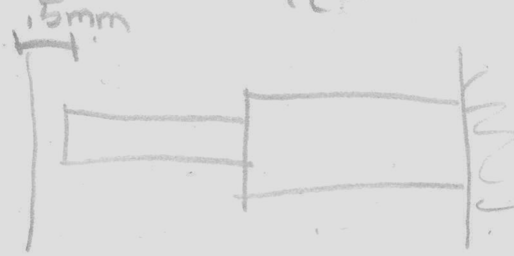
→ To solve this problem I use the principle of superposition. This problem is statically Indeterminant if the bars stretch long enough to touch point D. We have Two unknowns (R_D, R_E) and 1 equation, ($\Sigma F_x = 0$). However, we are able to solve this relation now because we have learned about stress and strain and how forces (statics) are related to displacements (kinematics). This relationship provides the necesary equations to solve a statically indeterminant problem.

→ For the principle of linear superposition, I take advantage of the fact that everything is linear and so I can add results together and everything is still consistent. In this principle, I isolate each cause of stress/strain, or in other words, each temperature displacement or force, and solve for its individual Δ(displacement). I then combine them all together with a constraint equation (or Boundary condition). This constraint equation comes when I erase one of the boundary conditions. When I release a boundary condition, I must "replace" it with a Force and displacement constraint equations otherwise the system will not be accurately represented. In other words, the boundary condition @ Point D can produce a reaction (or force) and cannot move (displacement constraint). Since, I am →

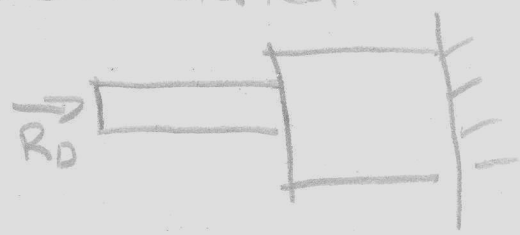
→?.. Continuation of problem 3

..., Since I am removing the support @ D and nothing in this world is free, I must account for the Force it could cause (R_d) and the displacement constraint it imposes or my System will not be consistent with the original problem. Once I have done this, I break everything apart and solve for individual displacements each caused by a force or change in temperature. I then plug that into my displacement constraint and use it to "enforce" reality (or the boundary conditions).

Let's Get Started



→



Displacement constraint → $\Delta_T = \Delta_{Temp} + \Delta_{R_D} \leq .5mm$

<u>Note</u>: if $\Delta_{Temp} \leq .5 \rightarrow \Delta_{R_D} = 0$

(I) 

$\Delta_{Temp\,Bronze} = \alpha_B \Delta T L_B = 7.56 \times 10^{-4}\,m$

(II) 

$\Delta_{Temp\,Aluminum} = \alpha_A \Delta T L_A = 10.44 \times 10^{-4}\,m$

(III) 

$\Delta_{R_D} = -\Delta_{R_{Bronze}} - \Delta_{R_{Alum}} = -f_{Bronze} R_D + f_{Alum} R_D$

$\Delta_{R_D} = -\left(\frac{L_B}{E_B A_B} + \frac{L_A}{A_A E_A}\right) R_D$

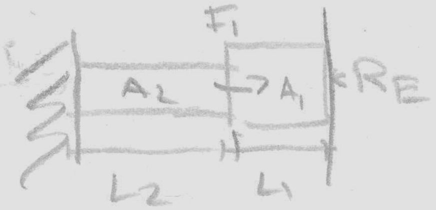
In compression so I know $\Delta_{R_D} \leq 0$

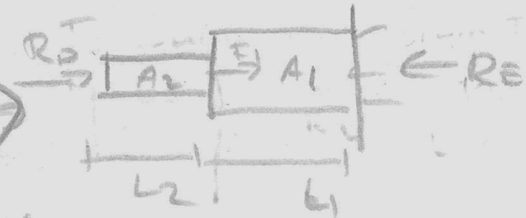
R_E always exists. R_E is what provides the counter force so that the shape is in equilibrium. This is why when using superposition the length the force "strains" is from the point the force acts to the support.

↳ See the next page for an example ↵

Brief Interlude to explain how Superposition accounts for strain.

If I have this!





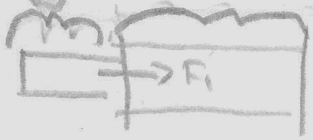
and $\Delta_{Total} = 0$

assume the same E.

no opposing forces to cause strain.

This part is in compression so only this part receives strain from F_1 while solving via the linear Superposition method.

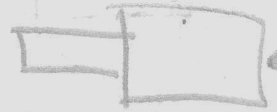
(I)



$$\Delta_1 = -\frac{F_1 L_1}{A_1 E}$$

In compression

(II)



This whole thing is now in compression

$$\Delta_2 = -\frac{R_D L_2}{A_2 E} - \frac{R_D L_1}{A_1 E}$$

In compression

$$\Delta_1 + \Delta_2 = 0$$

$$-\frac{F_1 L_1}{A_1 E} = R_D \left(\frac{L_2}{A_2 E} + \frac{L_1}{A_1 E} \right)$$

$$R_D = -\frac{L_1}{A_1} \left(\frac{A_2 A_1}{L_2 A_1 + L_1 A_2} \right)$$

Back To Solving:

$$(\Delta_{TempBronze} + \Delta_{TempAluminum}) + \Delta_{R_D} = .0005$$

Sub in values.

$$(-7.56 \times 10^{-4} + 10.44 \times 10^{-4}) - R_D (2.22 \times 10^{-9} + 3.42 \times 10^{-9}) = .0005$$

$$-R_D (5.64 \times 10^{-9}) = -.0013$$

$$R_D = 230 \text{ kN}$$

next page

Since $R_D = 230\,kN$

$\Sigma F_x = 0$

$R_E = R_D = 230\,kN$

b) compressive force of bar

$\boxed{P = 230\,kN \text{ compression}}$

* Note, I did not check this value, but the hw had $\Delta T = 95°$ and this Problem has $\Delta = 100°$ From hw $R_D = 217\,kN$, and for this problem $R_D = 230\,kN$ so I am pretty sure it is right.

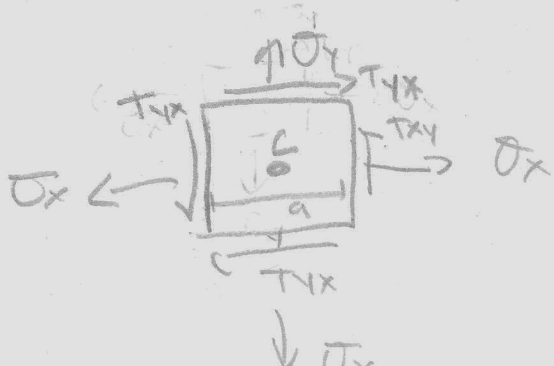
c) Stress:

$$\sigma_{bronze} = \frac{P_{bronze}}{A_{bronze}} = 153 \times 10^3\ KPa \quad \text{Compression}$$

$$\sigma_{Aluminum} = \frac{P_{Al}}{A_{Al}} = 128 \times 10^3\ KPa \quad \text{Compression}$$

Problem 4

A. Possible Equilibrium Stress States



$\Sigma M_C = 0$

$\tau_{yx}(a) - \tau_{xy}(a) = 0$

$\tau_{yx} = \tau_{xy}$ ✓

Stress tensor:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

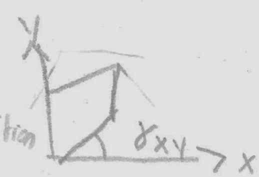
$\tau_{yx} = \tau_{xy}$ So the matrix must be symmetric for equilibrium, σ_y and σ_x equilibrate themselves

$\boxed{\text{b and d}}$

B) Shear strain:

a) relative elongation (that's normal stress WRONG!)

b) change of angle (Yes) → definition

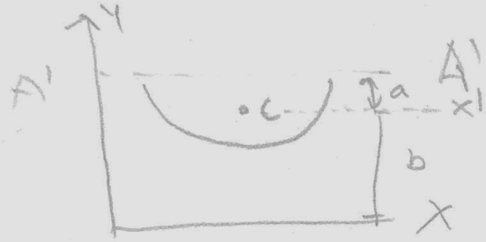


c). has nothing to do with temperature (correct)

d) has something to do with change in shape (correct) ↳ angle changes

e) something to do w/ Volume (Wrong! shape changes not volume)

C.



$I_{AA'} = \pi r^4/8$

Parallel axis theorem only works about the centroid!
As a result we need I_{cx}

$I_x = I_{cx} + Ad^2$

$I_{AA'} = I_{cx} + Aa^2$

$I_{cx} = I_{AA'} - Aa^2$

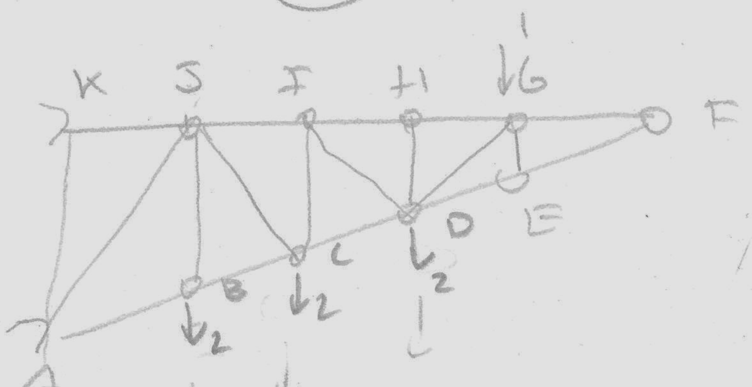
$I_{xx} = I_{cx} + Ab^2$

$I_{xx} = I_{AA'} - Aa^2 + b^2(A)$

$$\boxed{I_{xx} = \frac{\pi r^4}{8} - a^2A + b^2A}$$

d

D)

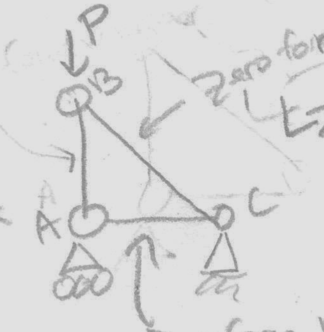


Zero force member occur when two members come together and there is no reaction or external force to oppose the forces from a member or when 3 members come together and two of them are in the same plane and there is no external force or reaction to oppose one of the members forces

Examples of zero force members:

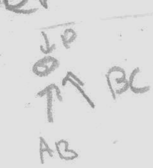
zero force member

ex



Zero force member

↳ @ B



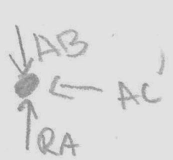
$\Sigma F_x = 0$

$BC \cos\theta = 0$

$ABc = 0$

Zero force member

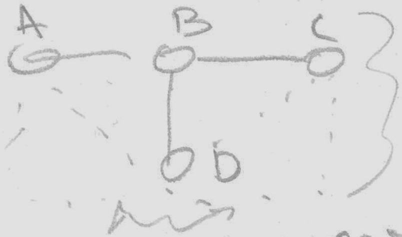
↳ @ A



$\Sigma F_x = 0$

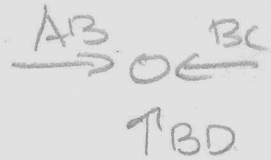
$Ac = 0$

Another example of zero force.



3 members (let's imagine this is a small part of a truss)

The rest of the truss structure is not shown besides all the members that connect @ B.



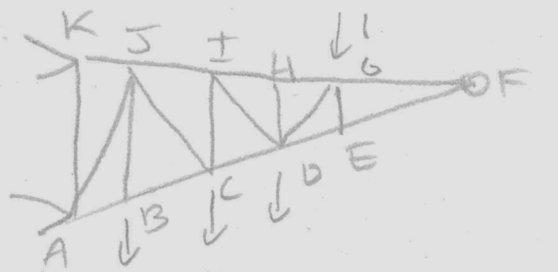
$\Sigma F_x = 0$

$AB = BC$

$\Sigma F_y = 0$

$BD = 0$ ← Zero Force member

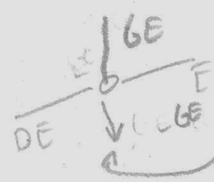
You should be able to just see this on the test! (no $\Sigma F_x = 0$, $\Sigma F_y = 0$)



From observation:

→ GF, FE are zero force members

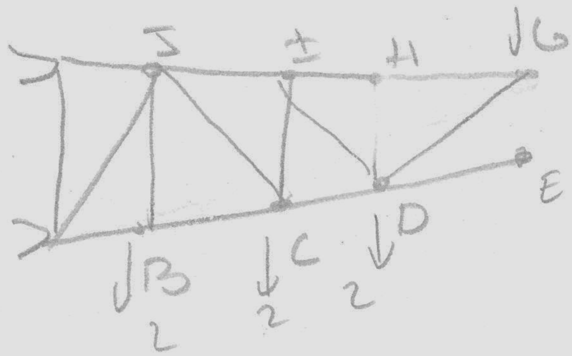
→ GE is a zero force member



can push in this direction and EF/DE cannot resist perpindicular forces.

→ HD is a zeroforce member

Let's redraw:



→ DE is dangling → zero force

→ cannot find anymore (DE does not cause any other zero force members)

∴ GF, FE, GE, HD, DE are zero force

So

$\boxed{a, b\ d, e\ f}$