

Problem 1

a)

The heat the ice absorbs in melting and getting to the same temperature as the water is

$$Q_{ice} = ML + Mc\Delta T = 80000 \text{ cal} + 1000T_f \text{ cal}/^\circ\text{C}.$$

By conservation of energy, this must be the heat given off by the water cooling

$$Q_{water} = Mc\Delta T = 1000 \text{ cal}/^\circ\text{C}(100^\circ\text{C} - T_f)$$

Setting the two equal gives

$$T_f = \frac{80000 \text{ cal} - 100000 \text{ cal}}{-2000 \text{ cal}/^\circ\text{C}} = 10^\circ\text{C}$$

b)

The entropy change of the ice is

$$S = \frac{ML}{T} + MC \int_{273\text{K}}^{283\text{K}} \frac{dT}{T} = \frac{80000 \text{ cal}}{273 \text{ K}} + 1000 \ln\left(\frac{283}{273}\right) \frac{\text{cal}}{\text{K}} \approx 330 \frac{\text{cal}}{\text{K}}.$$

(I used $\ln(1+x) \approx x$ to get a value for the logarithm). The entropy change of the water is

$$S = MC \int_{373\text{K}}^{283\text{K}} \frac{dT}{T} = 1000 \ln\left(\frac{283}{373}\right) \frac{\text{cal}}{\text{K}} \approx -280 \frac{\text{cal}}{\text{K}}$$

(I used $\ln(1+x) \approx x - x^2/2$ to get a value for the logarithm, since this is a bit further from 1). So the total entropy change is roughly $50 \frac{\text{cal}}{\text{K}}$. As expected from the second law, it is positive.

Problem 2

a)

The cycle is a heat engine when the integral around the whole cycle is positive. This happens if the cycle in run clockwise.

b)

The total work is the sum of the work along the isotherms

$$W = nR(T_1 - T_2) \ln\left(\frac{V_2}{V_1}\right).$$

The heat absorbed, Q_h is

$$Q_h = nC_v(T_1 - T_2) + nRT_1 \ln\left(\frac{V_2}{V_1}\right).$$

$C_v = \frac{3}{2}R$, as this is a monatomic gas. So the efficiency is

$$e = \frac{W}{Q_h} = \frac{nR(T_1 - T_2) \ln\left(\frac{V_2}{V_1}\right)}{n\frac{3}{2}R(T_1 - T_2) + nRT_1 \ln\left(\frac{V_2}{V_1}\right)} = \frac{(T_1 - T_2) \ln\left(\frac{V_2}{V_1}\right)}{\frac{3}{2}(T_1 - T_2) + T_1 \ln\left(\frac{V_2}{V_1}\right)} = \frac{1 - \frac{T_2}{T_1}}{1 + \frac{3}{2} \frac{1 - \frac{T_2}{T_1}}{\ln\left(\frac{V_2}{V_1}\right)}}$$

Which is clearly less efficient than a Carnot engine.

Problem 3

a)

Suppose the particle has just hit the wall. If the particle has a speed v , this means it will get to the wall again once it has traveled a distance $2L$. Thus, the time between collisions with a wall is $\Delta t = \frac{2L}{v}$. Since the gas molecules do not interact with one another and the collisions with the wall are elastic, v is a constant.

b)

When the particle collides with the wall, its momentum changes by $\Delta p = m\Delta v$. Since the collisions are elastic, $\Delta v = 2v$. Thus, the average force a particle exerts is

$$\bar{F}_1 = \frac{\Delta p}{\Delta t} = \frac{2mv^2}{2L} = \frac{mv^2}{L}.$$

The box contains many particles, each of which contribute this much force. Thus, the average total force due to all particles is

$$\bar{F} = \sum_{i=1}^N \frac{mv_i^2}{L} = \frac{m}{L} \sum_{i=1}^N v_i^2.$$

Here, v_i is the speed of the i th particle. The sum is just the definition of the average of the squared velocities times the number of particles. Also, in the limit that N is large, the timescales that F fluctuates on will be so small that they will not be detectable. Thus

$$F = \frac{mN}{L} \bar{v}^2.$$

c

Plugging in the definition gives

$$F = \frac{Nk_B T}{L}$$

So the equation of state is

$$FL = Nk_B T$$

You may notice this looks a lot like the ideal gas law!

Problem 4

The first thing we want to recognize is that the processes occurring during the transformation are adiabatic processes of ideal gases. The relevant equations will then be

$$PV = nRT \quad (1)$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \quad (2)$$

$$P_i V_i^\gamma = P_f V_f^\gamma. \quad (3)$$

The first equation is the ideal gas law, and the second and third (actually just two presentations of the same equation) give the relationships between the state variables during an adiabatic process. The exponent γ is related to the number f of quadratic degrees of freedom by $\gamma = (f + 2)/f$. The gas in container A is diatomic, so $f_A = 5$ and $\gamma_A = 7/5$. The gas in container B is monatomic, so $f_B = 3$ and $\gamma_B = 5/3$.

Throughout the solution, we'll refer to the initial and final pressures of A and B by P_{Ai} , P_{Af} , P_{Bi} , and P_{Bf} , respectively, and similarly for temperature and volume.

A.

Let's start with gas B . We know its initial and final temperatures, and we know its initial pressure, so we can use the ideal gas law to find its initial volume:

$$V_{Bi} = \frac{nRT_{Bi}}{P_{Bi}} = \frac{nRT_0}{P_0}. \quad (4)$$

Then we can use equation (2) to find its final volume:

$$T_{Bi} V_{Bi}^{2/3} = T_{Bf} V_{Bf}^{2/3} \longrightarrow V_{Bf} = V_{Bi} \left(\frac{T_{Bi}}{T_{Bf}} \right)^{3/2} = \frac{nRT_0}{P_0} \left(\frac{T_0}{2T_0} \right)^{3/2} = \frac{1}{2\sqrt{2}} \frac{nRT_0}{P_0}. \quad (5)$$

Now we turn to gas A . We don't know its final temperature, so using equation (2) doesn't look feasible. What about equation (3)? We know the initial pressure and volume, but we don't know the final pressure. However, we are told that the divider is free to move. This means that the two gases should be in mechanical equilibrium. In other words, their final pressures are equal, and we can use the ideal gas law to determine the final pressure of gas B , given that we know its final temperature and volume. Then:

$$P_{Af} = P_{Bf} = \frac{nRT_{Bf}}{V_{Bf}} = \frac{2nRT_0}{nRT_0} \cdot 2\sqrt{2}P_0 = 4\sqrt{2}P_0. \quad (6)$$

Now we can use equation (3):

$$P_{Ai} V_{Ai}^{7/5} = P_{Af} V_{Af}^{7/5} \longrightarrow V_{Af} = V_{Ai} \left(\frac{P_{Ai}}{P_{Af}} \right)^{5/7} = \left(\frac{nRT_0}{P_0} \right) \left(\frac{P_0}{4\sqrt{2}P_0} \right)^{5/7} = 2^{-25/14} \frac{nRT_0}{P_0} \quad (7)$$

where we again used the ideal gas law to find the initial volume.

B.

Because all processes in this transformation are adiabatic ($Q = 0$), the first law of thermodynamics simplifies:

$$\Delta E_{\text{int}} = Q + W = W, \quad (8)$$

where we've reversed the sign convention of work because the problem asks for work done *on*, rather than by the piston. But ΔE_{int} is simple to find:

$$\Delta E_{\text{int}} = \frac{f}{2} nR(T_f - T_i). \quad (9)$$

The change in the internal energy of gas B is then just

$$\Delta E_B = \frac{3}{2}nR(2T_0 - T_0) = \frac{3}{2}nRT_0. \quad (10)$$

To find the change in energy of gas A , we need the final temperature. We have everything we need to plug into the ideal gas law to find it:

$$T_{Af} = \frac{P_{Af}V_{Af}}{nR} = \frac{(4\sqrt{2}P_0)(2^{-25/14}nRT_0/P_0)}{nR} = 2^{5/7}T_0. \quad (11)$$

The change in energy is then

$$\Delta E_A = \frac{5}{2}nR(2^{5/7}T_0 - T_0) = \frac{5}{2}(2^{5/7} - 1)nRT_0. \quad (12)$$

Adding the contributions from A and B , we find the total change in energy, which is equal to the total work done on the piston:

$$W = \left(\frac{3}{2} + 5 \cdot 2^{-2/7} - \frac{5}{2}\right)nRT_0 = \left(5 \cdot 2^{-2/7} - 1\right)nRT_0 \quad (13)$$

Problem #5

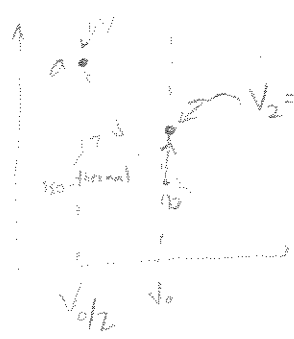
(a) The change in internal energy of the system is due entirely to relaxation of the springs, since there is no energy ~~change in~~ the gas as it undergoes adiabatic free expansion.

In this case

$$\Delta U = \int_{\substack{\text{line of spring} \\ \uparrow}} \vec{F} \cdot d\vec{x} = \frac{Bx^2}{2} = \frac{K(\Delta L)^2}{2}$$

displacement of piston

(b) We can get the change of entropy by looking at initial and final states of the system:

$$V_1 = V_0/2, P_1 = \frac{K(\Delta L)}{S} = \frac{\text{force of compressed spring}}{\text{surface area of piston}}$$


$V_2 = V_0, P_2$ found using ΔU :

$$\Delta U = \frac{3}{2} nR \Delta T \Rightarrow \Delta T = \frac{K(\Delta L)^2}{3nR}$$

$$T_2 = T_1 + \frac{K(\Delta L)^2}{3nR}$$

$$= \frac{K(\Delta L) V_0}{2nRS} + \frac{K(\Delta L)^2}{3nR}$$

$$= \frac{K(\Delta L)}{nR} \left(\frac{V_0}{2S} + \frac{\Delta L}{3} \right)$$

$$P_2 = \frac{K(\Delta L)}{S} \left(\frac{1}{2} + \frac{\Delta L}{3V_0} \right)$$

Entropy change is given by the path drawn above:

$$\Delta S = \Delta S_{ab} + \Delta S_{bc}$$

$$= \int_a^b \frac{dQ}{T} + \int_b^c \frac{dQ}{T}$$

\downarrow $K_0 = T$ \downarrow $nC_v = V$

$$= \frac{W_{ab}}{T} + \int_{T_b}^{T_c} \frac{nC_v dT}{T}$$

$$W_{ab} = \int \frac{nRT}{V} dV$$

$$= \frac{nRT \ln(V_0/(V_0/2))}{T} + \frac{3nR}{2} \ln\left(\frac{T_c}{T_b}\right) = \left(nR \ln(2) + \frac{3nR}{2} \ln\left(1 + \frac{\Delta L}{3} \left(\frac{2S}{V_0}\right)\right) \right)$$