MIDTERM 1 Fall-2014

Instructor: Prof. A. LANZARA

TOTAL POINTS: 100

Show all work, and take particular care to explain what you are doing. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make.If you get stuck, skip to the next problem and return to the difficult section later in the exam period.
Your solutions should show a logical progression of steps. Start with equations on the equation sheet.

Please use one sheet per problem and have them appear in order in your green book. If you need extra room for a problem, place it at the end, after all of the other problems and include a note in the original problem to guide the grader there.

GOOD LUCK!

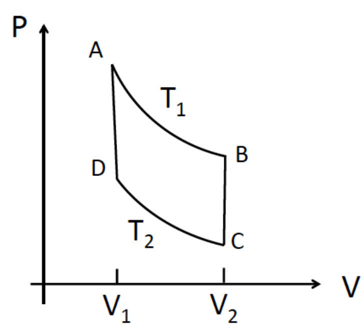
PROBLEM 1 (tot 20pts)

A thermally isolated container contains a mass M=1kg of ice at temperature T=0°C. An equal mass of water at temperature T=100°C is added to the container. The pressure P of the water-ice system is 1atm.
The specific heat of the water doesn't depend on the temperature and is equal to c_w=1cal/(gr °C). The latent heat of fusion of ice is equal to L = 80 cal/gr. Neglect the change of volume of ice and water and the specific heat of the container. Find:

a. (10pts) The final equilibrium temperature
b. (10pts) The change of entropy of the water-ice system between the initial and final state

PROBLEM 2 (tot 20pts)

A thermal engine operates between two reservoirs T_1 and T_2 and utilizes n moles of an ideal monatomic gas. The engine follows the cycle shown below, made of a reversible isothermal expansion, cooling at constant volume; an isothermal compression and heating at constant volume.



a. (5pts) Which way should the cycle operate such that the engine is a thermal engine that delivers work?
b. (15pts) Determine the efficiency of the engine

PROBLEM 3 (tot 20pts)

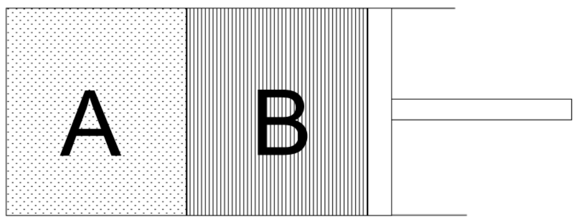
Suppose N gas molecules each with mass m live on a line, meaning they can only move left or right. The gas molecules do not interact with one another, but are confined to a (1D) box with length L. When the gas molecules reach the walls, they elastically collide and change direction.

a. (3pts) If a molecule has speed v, how often will it collide with the wall?
b. (12pts) How much force, F, needs to be applied at the end of the box to balance the force of the gas molecules on the end of the box?
c. (5pts) Define the temperature, T, of the gas as $m\overline{v^2}/k_B$ (this is just the equipartition theorem). What is the equation of state for this gas?

PROBLEM 4 (tot 20pts)

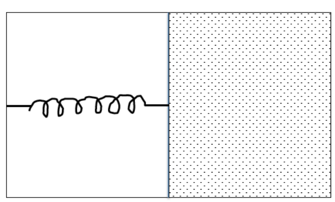
A thermally isolated container is divided in two equal parts A and B (see figure below). The divider between the gasses is free to move and transmits no heat. A contains n moles of an ideal diatomic gas at temperature T_0 and pressure P_0. B contains n moles of an ideal monatomic gas at the same pressure and temperature. We now compress the system in a reversible way by pushing the piston until the gas in B reaches a temperature $2T_0$.

a. (10 pts) What is the final volume of A and B?
b. (10 pts) Find the work applied on the piston.



PROBLEM 5 (tot 20pts)

A cylindrical container of volume V_0 is divided in two equal parts by a rigid piston of section S and negligible volume. The piston is kept in equilibrium by a spring of elastic constant K. The spring is initially compressed by ΔL (see figure below).
n moles of an ideal monatomic gas are contained on the right side of the container. The left side of the container is empty.



A tiny aperture is now opened on the piston, and as a result the gas will diffuse very slowly to the left side of the container. After the system has reached equilibrium, find:

a. (10pts) The change in internal energy of the gas
b. (10pts) The change of entropy of the system

Note: Neglect the heat capacity of the container, piston and spring and treat the motion of the piston as frictionless.

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$E_{int} = \frac{d}{2}NkT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$\Delta E_{int} = Q - W$$

$$dE_{int} = dQ - PdV$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For a reversible adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2}R$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$\text{COP} = \frac{Q_L}{W}$$

$$S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0 \text{ (For irreversible processes)}$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

	Q	W
Isobaric	$C_P n\Delta T$	$P\Delta V$
Isochoric	$C_V n\Delta T$	0
Isothermal	$nRT\ln\left(\frac{V_f}{V_0}\right)$	$nRT\ln\left(\frac{V_f}{V_0}\right)$
Adiabatic	0	$-\frac{d}{2}(P_f V_f - P_0 V_0)$