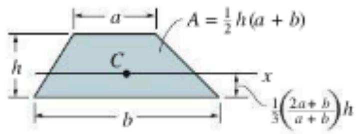
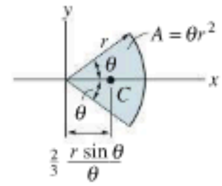
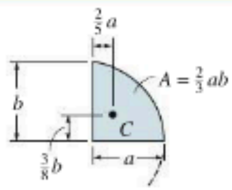
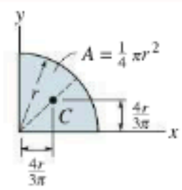
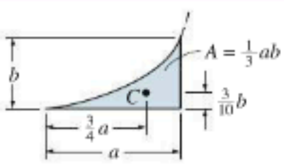
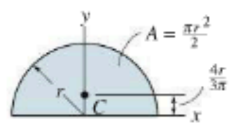
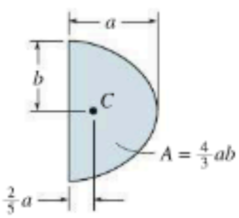
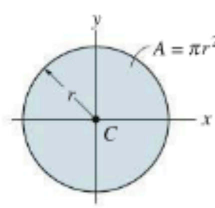
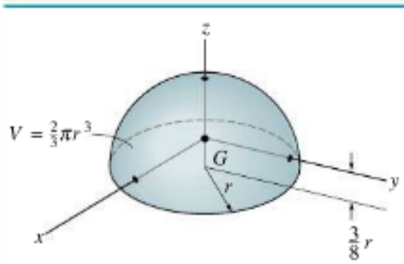
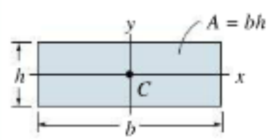
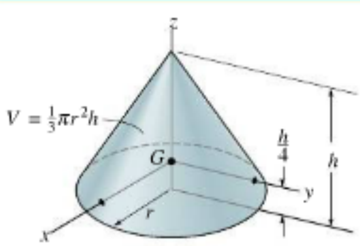
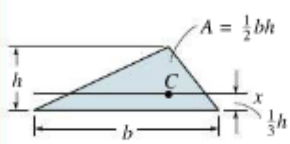
Introduction to Solid Mechanics
ME C85/CE C30

Final Exam

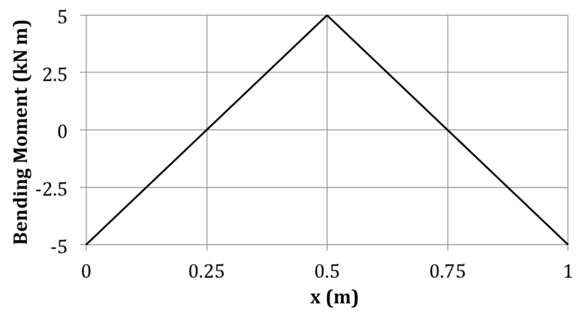
Fall, 2013

1. Leave an empty seat between you and the person (people) next to you. Unfortunately, there have been reports of cheating on the midterms, so we will be monitoring activity in the exam room closely during this exam.
2. Do not open this exam until you are told to begin.
3. Before beginning to solve the problems, please review the contents of the first few pages and the last few pages as they contain material that may be useful during the exam.
4. Put your name and SID on **every** page of the answer booklet.
5. You may not use a calculator, but you may use a straightedge to help you draw figures.
6. You may use two 8-1/2 x 11 sheet of notes, but not your book or any other notes.
7. Store everything else out of sight.
8. Turn off cell phones.
9. There will be no questions during the exam. Write your concerns or alternative interpretations in exam margins.
10. Be concise and write clearly. Identify your answer to a question by putting a box around it.
11. Use only the front sides of the answer sheets for your answers. You may use the backs of pages for "scratch" paper, but if there is work that we should see, be sure to point that out in the main body of the exam.
12. Time will be strictly enforced. At 10:00, you must put down your pencil or pen and immediately turn in your exam. Failure to do so may result in loss of points.

Centroid Location	Centroid Location	Area Moment of Inertia
 Trapezoidal area	 Circular sector area	$I_x = \frac{1}{4}r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{1}{4}r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Semiparabolic area	 Quarter circular area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Exparabolic area	 Semicircular area	$I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
 Parabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
 Hemisphere	 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Cone	 Triangular area	$I_x = \frac{1}{36}bh^3$

Problem 1. (20 Points)

Consider a 1m long beam supported at its ends either by pinned or fixed supports. The type of support can be different at each end. Shown at the right is the bending moment diagram for this beam.



From the information given:

(a) Determine the type of the support (fixed or pinned) at each end of the beam. Justify your answers.

(b) Draw the associated shear diagram for the beam.

(c) Sketch the beam, showing the supports and the applied loading

(d) Determine the equation for the slope of the beam $\theta(x)$ between $x = 0$ and $x = 0.5$, assuming that you know the value of Young's modulus E and moment of inertia I. (You may express your answer with EI as a factor in the appropriate location(s).)

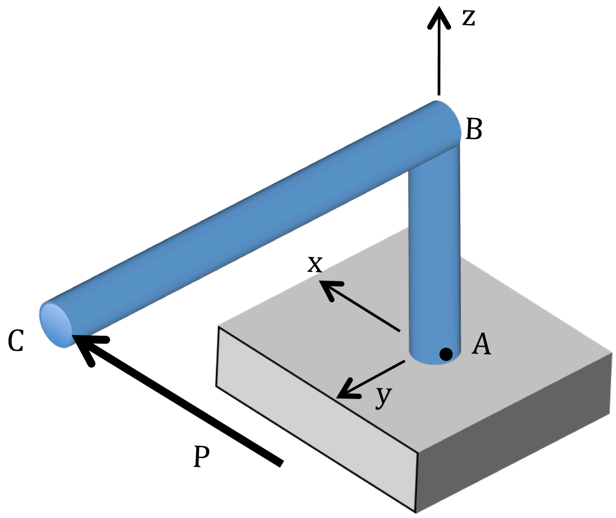
(e) Sketch the slope of the beam between $x = 0$ and $x = 0.5$. Explain why your curve looks the way that it does. Identify any points of particular interest (inflection points, maximum or minimum values, end values) in light of the preceding information and work.

(f) Determine the elastic curve giving the deflection of the beam $v(x)$ between $x = 0$ and $x = 0.5$, assuming that you know the value of Young's modulus E and moment of inertia I. (You may express your answer with EI as a factor in the appropriate location(s).)

(g) Sketch the deflected shape of the beam between x=0 and x=0.5. Explain why your curve looks the way that it does. Identify any points of particular interest (inflection points, maximum or minimum values, end values) in light of the preceding information and work.

Problem 2. (20 Points)

Member ABC has a uniform circular cross section with radius r. It is built into a rigid base at A and is bent through a 90 degree angle at B. Segment AB is vertical (along the z-axis) and is of length L, while segment BC is horizontal (parallel to the y-axis) and is of length $2L$. The material is elastic, with Young's modulus E and shear modulus $G = 2E/5$ (this corresponds to a Poisson's ratio of 0.25). Force P is horizontal (parallel to the x-axis) and acts at the free end at point C.



(a) Determine the deflection of point C. (You may assume small deformations so that the deflection of point C is along the line of action of P.)

(b) Determine the components of stress at the point on the surface of member AB located at $x = -r,\ y = 0,\ z = 0$. This point is identified by the "dot" on the surface of the member. Note: this point is in plane stress, with $\sigma_x = 0,\ \tau_{xy} = 0, \tau_{xz} = 0$.

(c) Determine the principal stresses at this point.

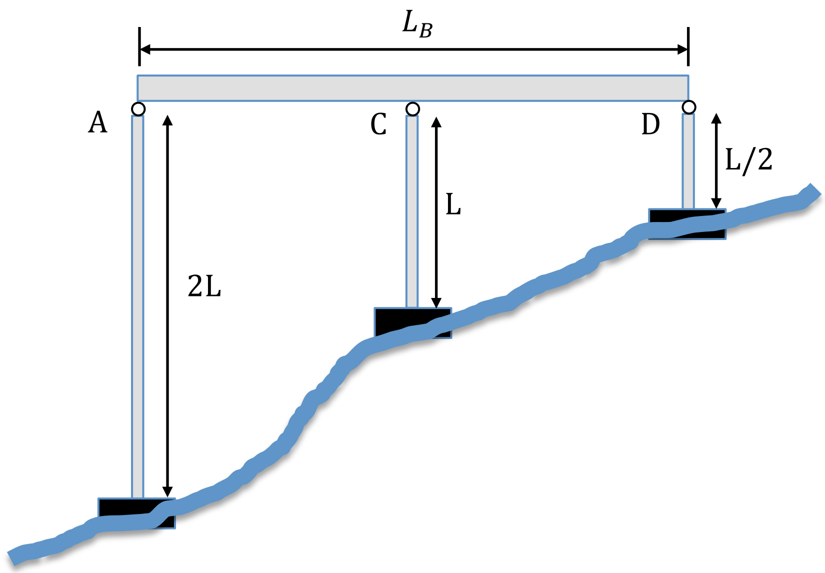
Problem 3. (20 Points)

This problem considers a portion of a bridge with total length L_B between two expansion joints. We will treated the bridge as an elastic beam with Young's modulus E_B and moment of inertia I_B. Its weight per unit length is w. The section of interest is supported by three columns with lengths $2L$, L, and $L/2$.

As a first approximation to the forces acting on each column, we will consider them to be rigid so that they provide, effectively, pinned supports at the ends and at the midpoint of the bridge section. Each column is built in (fixed) at its base.

(a) Determine the force applied at each support (at A, C and D). Since you are not allowed to use a calculator, it may be useful to know that $48 = 3 * 2^4$ and $384 = 3 * 2^7$.

(b) Now we treat the columns as elastic, with Young's modulus E, cross sectional area A and moment of inertia I, and let each be acted upon by the force determined in part (a) of this problem. Assuming that failure will occur by buckling of one of the columns, determine the maximum weight per unit length w that can be supported. Identify which is the limiting column (i.e., the column that buckles first).

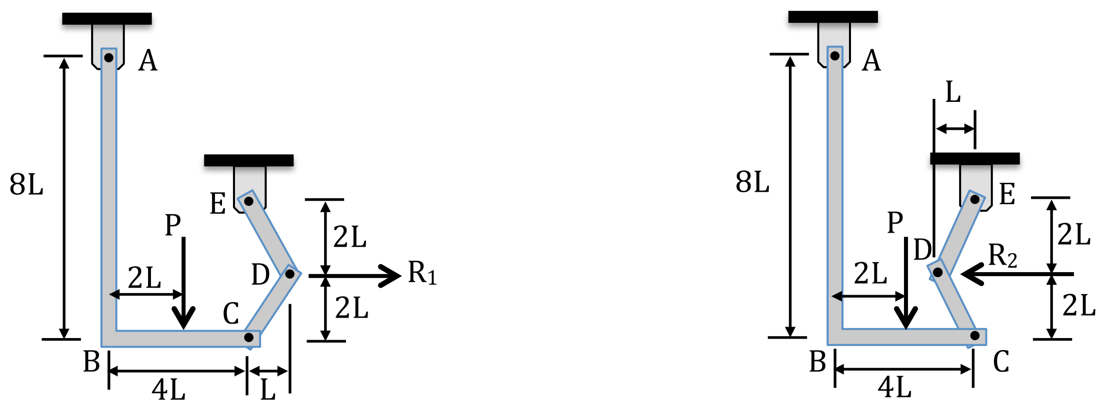


Problem 4. (20 Points)

Two similar frames are shown below. For each, force P is applied at the midpoint of the horizontal section of massless, rigid member ABC. Members CD and DE may also be treated as massless and rigid. The frames are held in their respective shapes by horizontal forces R_1 and R_2 applied to point D.

(a) Show that for the frame on the left, the force required to maintain the configuration shown is $R_1 = P/4$. Note: be sure to identify and take advantage of any two-force members that may be present.

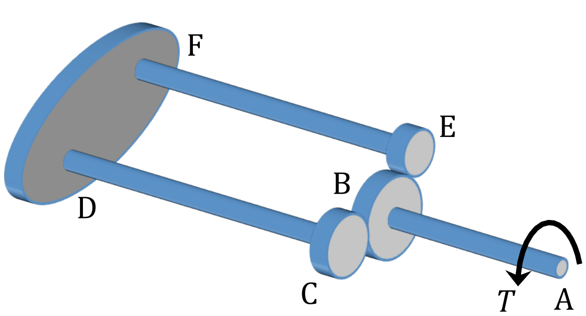
(b) Determine the force R_2 necessary to hold the frame on the right in place. Discuss this (perhaps surprising) result.

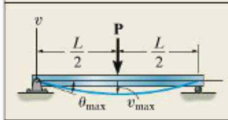
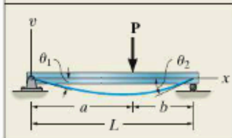
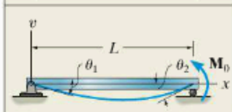
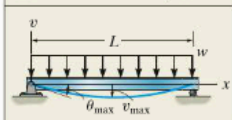
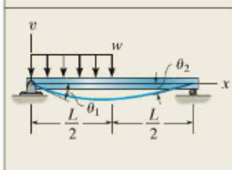
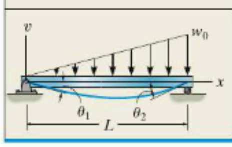


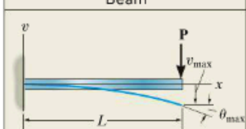
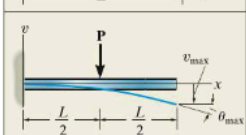
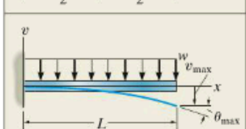
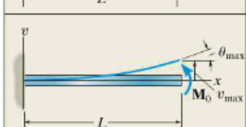
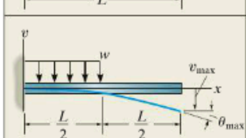
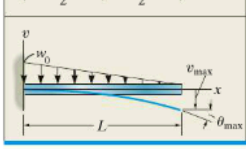
Problem 5. (20 Points)

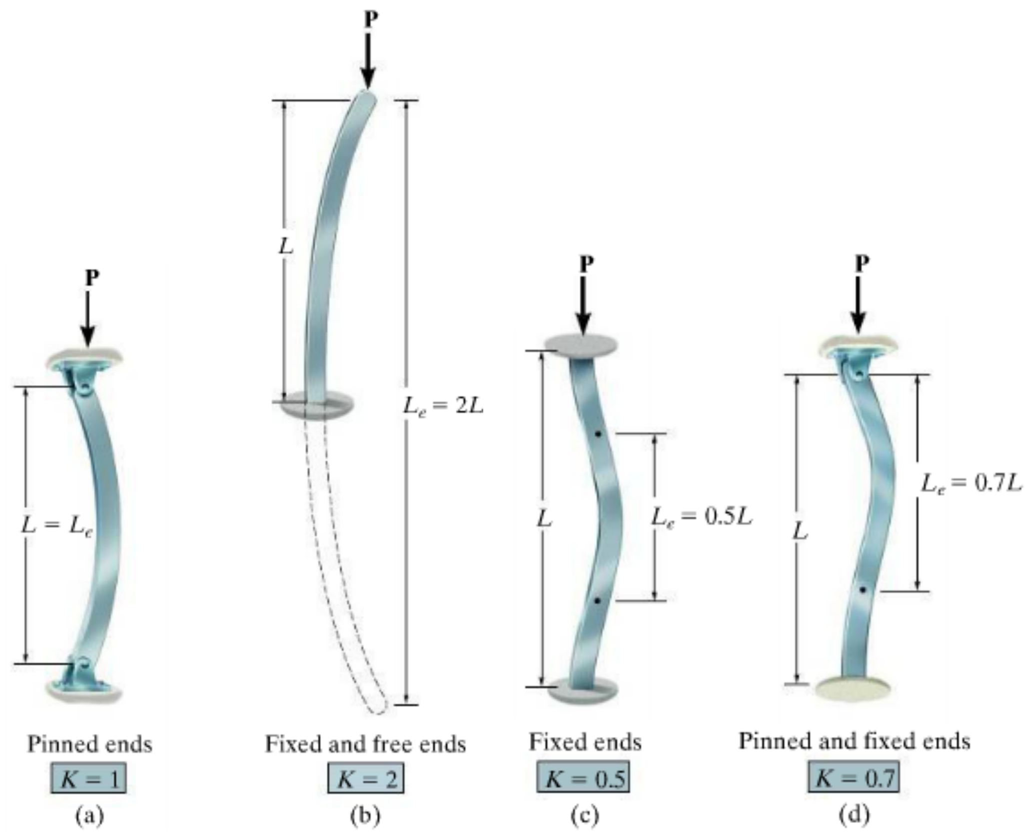
Shown below is a system that has gear B (with radius r_B) meshing with gears C and E (with radii r_C and r_E). Shafts CD and EF are identical, with length L, shear modulus G and polar moment of inertia J. The ends of the shafts at D and F are fixed.

What torque T must be applied at point A in order for gear B to rotate through an angle ϕ_B?



Simply Supported Beam Slopes and Deflections			
Beam	**Slope**	**Deflection**	**Elastic Curve**
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0L}{6EI}$ $\theta_2 = \dfrac{M_0L}{3EI}$	$v_{max} = \dfrac{-M_0L^2}{9\sqrt{3}EI}$ at $x = 0.5774L$	$v = \dfrac{-M_0x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0L^3}{360EI}$ $\theta_2 = \dfrac{w_0L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0L^4}{EI}$ at $x = 0.5193L$	$v = \dfrac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

Cantilevered Beam Slopes and Deflections			
Beam	**Slope**	**Deflection**	**Elastic Curve**
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{12EI}(3L - 2x)$ $\quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{48EI}(6x - L)$ $\quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0L}{EI}$	$v_{max} = \dfrac{M_0L^2}{2EI}$	$v = \dfrac{M_0x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{384EI}(8x - L)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$



Plane Stress: Principal and Maximum Shear Stresses

$$\sigma_{max}, \sigma_{min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$