

Stat 134 Midterm

Instructor: Mike Leong

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Name: _____

SID: _____

To get credit for work, you **must** show your work. Box in your final answer.

No calculators are allowed. So you may leave your answer as a numerical expression. Examples are listed below.

$$123^2 + 17^3 e^2$$
$$\Phi\left(\frac{123 - 100}{17}\right)$$

All summations must be evaluated for full credit. Scan through the test first to see which problems are easier for you. Manage your time.

#	Your Score	Points Possible
1		10
2		10
3		10
4		10
5		10
Total		50

(Blank for scratch work.)

1. Suppose the X_i 's are all independent with the following distribution.

$$X_i \sim \text{Pois} \left(\lambda_i = 81 \left(\frac{1}{3} \right)^i \right)$$

a) Let $S_4 = X_1 + \cdots + X_4$. Find $P(S_4 = 50)$.

(5 pt)

b) Let $S = \lim_{n \rightarrow \infty} (X_1 + \cdots + X_n)$. Find $E(S^2)$.

(5 pt)

2. Suppose $X \sim \text{Geom}(p_1)$ on $\{1, 2, 3, \dots\}$, $Y \sim \text{Geom}(p_2)$ on $\{1, 2, 3, \dots\}$, and $X \perp Y$. Let $S = X + Y$.

- a) Assuming $p_1 = p_2 = p$, find $P(S = s)$. (2 pt)
- b) Assuming $p_1 \neq p_2$, find $P(S = s)$. (4 pt)
- c) Find $P(Y \geq X)$. (4 pt)

3. A bag contains 3 types of coins.

(hh)	(ht)	(tt)
7 double headed coins	2 regular coins	1 double tailed coin

Select a coin from the bag and flip it twice. Let H_i be the event the i^{th} toss lands heads. Find:

- a) $P(H_1)$ (3 pt)
- b) $P(H_2)$ (1 pt)
- c) $P(H_2|H_1)$ (4 pt)
- d) Are H_1 and H_2 independent? Justify your answer. (2 pt)

4. Suppose $X \sim \text{Pois}(\mu)$, $Y \sim \text{Geom}(p)$ on $\{0, 1, 2, \dots\}$, and $X \perp Y$.

a) Find $P(X \geq 1)$.

(2 pt)

b) Find $P(Y \geq y)$.

(3 pt)

c) Find $P(Y \geq X)$.

(5 pt)

5. In a class of 85 students, let X be the number of students who share a birthday with at least two other members of the class.
- a) Write $E(X)$ as an unsimplified expression. (There should be no summation.) (5 pt)

- b) Estimate the probability using a Poisson distribution that at least 3 people have the same birthday. Your solution should not depend on part a). (5 pt)